

POLYNOMIAL INVARIANTS OF 2-BRIDGE KNOTS THROUGH 22 CROSSINGS

TAIZO KANENOBU AND TOSHIO SUMI

ABSTRACT. We calculate the homfly, Kauffman, Jones, Q, and Conway polynomials of 2-bridge knots through 22 crossings and list all the pairs sharing the same polynomial invariants.

1. INTRODUCTION

A simple question for polynomial invariants of knots is: “How many knots do they classify?” Concerning this problem, we made a computer experiment with 2-bridge knots, which are completely classified by Schubert [25]. We calculated the homfly, Kauffman, Jones, Q, and Conway polynomials of 2-bridge knots through 22 crossings, and searched all the pairs of 2-bridge knots having the same polynomial invariants. The total number of the knots is 350,207, where each chiral pair is counted as one knot. If a chiral pair is counted separately, then this amounts to 699,732. The program is written in Turbo Pascal for the NEC PC-9801 Series. In a sequel to this paper, we shall report on 2-bridge links.

The homfly polynomial $P_L \in Z[v^{\pm 1}, z^{\pm 1}]$ [6, 23] of an oriented link L is defined, as in [20], so that

$$v^{-1}P_{L_+} - vP_{L_-} = zP_{L_0},$$

where (L_+, L_-, L_0) is a skein triple. Putting $v = 1$, we get the Conway polynomial $\nabla_L \in Z[z]$ [3] (substituting $(v, z) = (1, t^{1/2} - t^{-1/2})$, we get the Alexander polynomial), and substituting $(v, z) = (t, t^{1/2} - t^{-1/2})$, we get the Jones polynomial $V_L \in Z[t^{\pm 1/2}]$ [8]. These are skein invariants. We refer to [18] for the definitions of skein triple and skein equivalence. The Kauffman polynomial $F_L \in Z[a^{\pm 1}, z^{\pm 1}]$ of an oriented link L is given by $F_L = a^{-w}\Lambda_D$, where Λ_D is the L-polynomial of a diagram D of L and w is the writhe of D . We refer to [13] for the definitions of a writhe and the L-polynomial. Putting $a = 1$, we get the Q polynomial $Q_L \in Z[z^{\pm 1}]$ [1, 7] of an unoriented link $|L|$, and substituting $(a, z) = (-t^{-3/4}, t^{-1/4} + t^{1/4})$, we get the Jones polynomial [16].

Received by the editor March 22, 1990 and, in revised form, July 8, 1991.

1991 *Mathematics Subject Classification*. Primary 57M25.

Key words and phrases. 2-bridge knot, homfly polynomial, Kauffman polynomial, Jones polynomial, Q polynomial, Conway polynomial.

The work of the first author was supported in part by Grant-in-Aid for Encouragement of Young Scientist (No. 01740057), Ministry of Education, Science and Culture.

If L is a 2-bridge knot or link, then we have

$$(1) \quad Q_L(z) = 2z^{-1}V_L(t)V_L(t^{-1}) + 1 - 2z^{-1},$$

where $z = -t - t^{-1}$ [11]. Thus, if we know the Jones polynomial of a 2-bridge knot or link, we can deduce the Q polynomial. In an early computer calculation of the polynomial invariants of 2-bridge knots and links, we found many pairs of 2-bridge knots and links with the same Q polynomial but distinct Jones polynomial, except for a reflection such as right- and left-handed trefoils. This has been generalized to the following theorem in [12]:

For any positive integer N , there exist N sets of 2^N 2-bridge knots S_1, S_2, \dots, S_N with $S_i = \{K_{i1}, K_{i2}, \dots, K_{i2^N}\}$ such that: all the knots in $\bigcup_{i=1}^N S_i$ have the same Q and Conway polynomials; all the knots in each S_i are skein equivalent; and all the knots $K_{11}, K_{21}, \dots, K_{N1}$ have mutually distinct Jones polynomials.

In addition, we observe the following for 2-bridge knots through 22 crossings:

Fact 1. $P_K(v, z) = P_{K'}(v, z)$ if and only if $V_K(t) = V_{K'}(t)$ and $\nabla_K(z) = \nabla_{K'}(z)$.

Fact 2. K is amphichiral if and only if $V_K(t) = V_K(t^{-1})$ (or $P_K(v, z) = P_K(v^{-1}, z)$).

Fact 3. The number of knots having the same homfly or Kauffman polynomial is at most two.

Regarding Fact 3, we can construct the following examples:

(i) Arbitrarily many 2-bridge knots with the same Jones polynomial ([9, Theorem 6]).

(ii) Arbitrarily many fibred, amphichiral, skein equivalent 2-bridge knots ([10, Theorem 1]).

(iii) A pair of fibred, amphichiral, skein equivalent 2-bridge knots with the same Kauffman polynomial ([10, Theorem 4]).

(iv) A pair of 2-bridge knots with the same Kauffman polynomial but distinct Alexander polynomials ([10, Theorem 5]).

Note that (ii) above does not necessarily include (i) because the 2-bridge knots constructed in (i) may have distinct Conway polynomials. For the Kauffman polynomial we shall give an example similar to (i) in a forthcoming paper.

2. FORMULAS

Let S_1 and S_2 be the elementary braids generating the 3-braid group as shown in Figure 1.

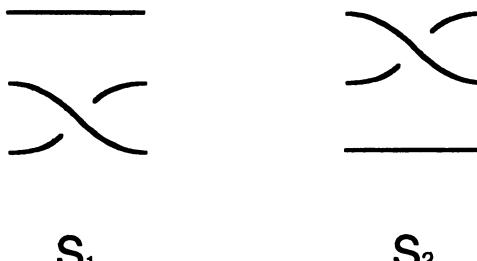


FIGURE 1

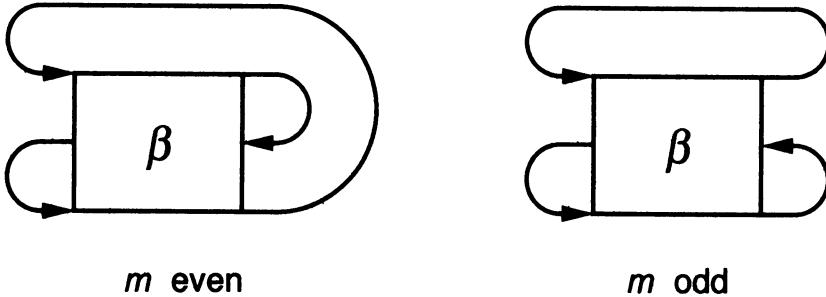


FIGURE 2

Let $D(b_1, b_2, \dots, b_m)$ be the oriented 2-bridge knot (m is even) or link (m is odd) with the corresponding diagram as shown in Figure 2. There, β is the 3-braid either $S_2^{2b_1} S_1^{-2b_2} \cdots S_1^{-2b_m}$ or $S_2^{2b_1} S_1^{-2b_2} \cdots S_2^{2b_m}$ depending on whether m is even or odd. Any 2-bridge knot or link can be put in this form.

Let $P(b_1, b_2, \dots, b_m)$, $V(b_1, b_2, \dots, b_m)$, $\nabla(b_1, b_2, \dots, b_m)$, $\Lambda(b_1, b_2, \dots, b_m)$, and $F(b_1, b_2, \dots, b_m)$ be the homfly, Jones, Conway, L, and Kauffman polynomials of $D(b_1, b_2, \dots, b_m)$, respectively.

Proposition 1 ([18, Proposition 14]). *There holds*

$$P(b_1, b_2, \dots, b_m) = (1, \mu) M(-b_1) M(b_2) \cdots M((-1)^m b_m) \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

where

$$M(b) = \begin{pmatrix} (1 - v^{2b})\mu^{-1} & 1 \\ v^{2b} & 0 \end{pmatrix}, \quad \mu = (v^{-1} - v)z^{-1}.$$

From this proposition, we have

Proposition 2 ([18, p.128]). *There holds*

$$\nabla(b_1, b_2, \dots, b_m) = (1, 0) N(-b_1) N(b_2) \cdots N((-1)^m b_m) \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

where

$$N(b) = \begin{pmatrix} bz & 1 \\ 1 & 0 \end{pmatrix}.$$

Proposition 3 (cf. [26]). *Let $\nabla_{-1} = 0$, $\nabla_0 = 1$, and $\nabla_m = \nabla(b_1, b_2, \dots, b_m)$ for $m \geq 1$. Then*

$$\nabla_m = (-1)^m b_m z \nabla_{m-1} + \nabla_{m-2}$$

for $m \geq 1$.

Therefore, if $b_i \neq 0$ for any i , then we have

$$(2) \quad \deg \nabla_m = m$$

for $m \geq 1$, and so the genus of $D(b_1, b_2, \dots, b_m)$ is either $m/2$ or $(m-1)/2$ according as m is even or odd [4, 21].

Proposition 4 ([17, Theorem 5]). *There holds*

$$\Lambda(b_1, b_2, \dots, b_m) = (1, a^{-1}, d) ST^{2b_1-1} ST^{-2b_2-1} S \cdots ST^{(-1)^{m-1} 2b_m-1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

where $d = (a + a^{-1})z^{-1} - 1$,

$$S = \begin{pmatrix} 0 & 0 & a \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } T = \begin{pmatrix} z & 1 & 0 \\ -1 & 0 & 0 \\ z & 0 & a \end{pmatrix},$$

and so

$$F(b_1, b_2, \dots, b_m) = a^{-w} \Lambda(b_1, b_2, \dots, b_m),$$

where $w = 2(-b_1 + b_2 - \cdots + (-1)^m b_m)$.

3. COMPUTATIONAL PROCESS

Step 1. Enumeration. We denote by $C(a_1, a_2, \dots, a_k)$ the unoriented 4-plat (or the unoriented diagram according to the context) as shown in Figure 3, where α is the 3-braid either $S_2^{a_1} S_1^{-a_1} \cdots S_1^{-a_k}$ or $S_2^{a_1} S_1^{-a_2} \cdots S_2^{a_k}$ according as k is even or odd.

An unoriented 2-bridge knot or link, or its mirror image, is uniquely represented as a 4-plat $C(a_1, a_2, \dots, a_k)$ satisfying the following conditions (3) and (4):

$$(3) \quad a_1, a_k \geq 2, \quad a_2, \dots, a_{k-1} \geq 1;$$

$$(4) \quad \text{either } a_i = a_{k-i+1} \text{ for all } i \geq 1, \text{ or } a_1 = a_k, a_2 = a_{k-1}, \dots, a_{i-1} = a_{k+2-i}, a_i > a_{k+1-i} \text{ for some } i \geq 1.$$

See [2, Proposition 12.13].

In order to enumerate all the 2-bridge knots and links of n crossings, we produce the sequences of integers $a_1 a_2 \cdots a_k$ satisfying (3),(4) and

$$(5) \quad a_1 + a_2 + \cdots + a_k = n.$$

See [14, 22, 27].

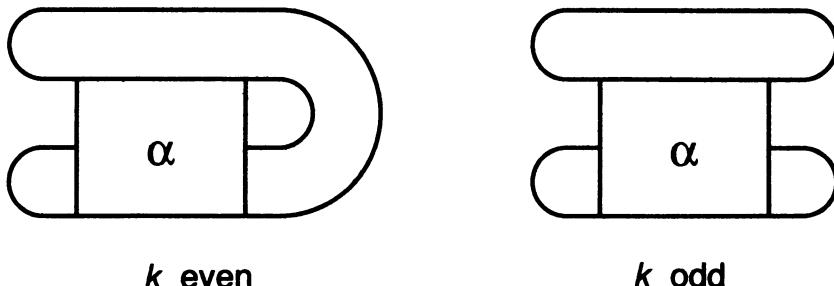
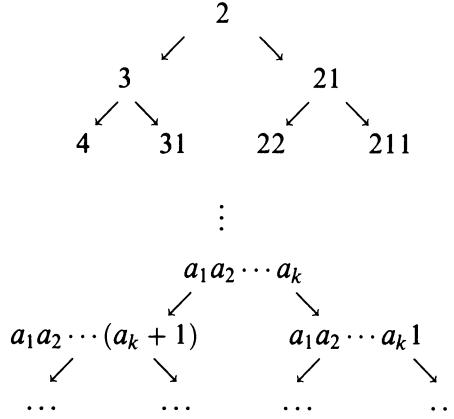


FIGURE 3

Specifically, we construct a binary tree as follows:



Then choose the sequences of integers satisfying (3)–(5) from the $(n - 1)$ st row. Calculate the coprime positive integers p and q by the continued fraction

$$(6) \quad \frac{p}{q} = a_1 + \frac{1}{a_2 + \cdots + a_k}.$$

The 2-fold covering space of S^3 branched over $C(a_1, a_2, \dots, a_k)$ is the lens space $L(p, q)$ [3]. See also [24, p. 303]. $C(a_1, a_2, \dots, a_k)$ is a 2-bridge knot if and only if p is odd; p is the determinant of the 2-bridge knot $C(a_1, a_2, \dots, a_k)$.

Take out the 2-bridge knots from the $C(a_1, a_2, \dots, a_k)$'s and order them as follows:

$$C(a_1, a_2, \dots, a_k) \prec C(a'_1, a'_2, \dots, a'_l)$$

if either $p < p'$ or $p = p'$ and $a_1 = a'_1, a_2 = a'_2, \dots, a_{i-1} = a'_{i-1}, a_i > a'_i$ for some i , where p and p' are the determinants of $C(a_1, a_2, \dots, a_k)$ and $C(a'_1, a'_2, \dots, a'_l)$, respectively.

Since a 2-bridge knot is invertible (cf. [2, Proposition 12.5]), we are not concerned with the question of knot orientation.

Let \mathcal{K}_n denote the ordered set of the 2-bridge knots $C(a_1, a_2, \dots, a_k)$ satisfying the conditions (3)–(5). Let

$$\overline{\mathcal{K}_n} = \{ C(-a_1, -a_2, \dots, -a_k) \mid C(a_1, a_2, \dots, a_k) \in \mathcal{K}_n \}.$$

Then the union $\mathcal{K}_n^* = \mathcal{K}_n \cup \overline{\mathcal{K}_n}$ is the set of all the 2-bridge knots of n crossings, and the intersection $\mathcal{A}K_n = \mathcal{K}_n \cap \overline{\mathcal{K}_n}$ is the set of all the amphichiral 2-bridge knots of n crossings. It is known [26] that $C(a_1, a_2, \dots, a_k)$ with (3)–(5) is amphichiral if and only if k is even and $a_i = a_{k+1-i}$ for all i . The numbers of these sets are explicitly given in [5].

Step 2. Calculation of the polynomial invariants. Let $K = C(a_1, a_2, \dots, a_k) \in \mathcal{K}_n^*$ and p, q be obtained from (6). If q is odd (resp. even), then let $(r, s) = (q-p, q)$ (resp. $(q, q-p)$). Then K is the 2-bridge knot with Schubert's normal form $S(p, s)$. The classification theorem states that $S(p_1, q_1)$ and $S(p_2, q_2)$ are isotopic if and only if $p_1 = p_2, q_1^{\pm 1} \equiv q_2 \pmod{p_1}$. Also,

K is isotopic to $D(b_1, b_2, \dots, b_m)$, where the b_i are obtained from the continued fraction

$$\frac{p}{r} = 2b_1 + \frac{1}{2b_2 + \dots + \frac{1}{2b_m}}.$$

Compute $P(b_1, b_2, \dots, b_m)$ and $F(b_1, b_2, \dots, b_m)$ using Propositions 1 and 4. Next compute $\nabla(b_1, b_2, \dots, b_m)$, $V(b_1, b_2, \dots, b_m)$, and $Q(b_1, b_2, \dots, b_m)$ by the substitutions as in the introduction. Note that $P_{\bar{K}}(v, z) = P_K(v^{-1}, z)$, $F_{\bar{K}}(a, z) = F_K(a^{-1}, z)$, $\nabla_{\bar{K}}(z) = \nabla_K(z)$, $V_{\bar{K}}(t) = V_K(t^{-1})$, and $Q_{\bar{K}}(z) = Q_K(z)$, where $\bar{K} = C(-a_1, -a_2, \dots, -a_k) \in \mathcal{K}_n$.

Step 3. Comparison of the polynomial invariants. We have searched for all pairs of 2-bridge knots through 22 crossings having the same polynomial invariant. We first considered the Q polynomial. Let K be as in Step 2. Since the crossing number n of K equals the degree of $Q_K(z)$ plus one [15, 19] and $Q_K(2) = p^2$ [1], we sought pairs having the same Q polynomial in the set $\mathcal{K}_{n,p} = \{K \in \mathcal{K}_n \mid \text{the determinant of } K \text{ is } p\}$ for each n and p . Let K_1 and K_2 be such a pair in $\mathcal{K}_{n,p}$. We sought pairs having the same Jones polynomial in $K_1, K_2, \bar{K}_1, \bar{K}_2$. To do so, we compared the four pairs: $\{V_{K_1}(t), V_{K_2}(t)\}$, $\{V_{K_1}(t), V_{K_2}(t^{-1})\}$, $\{V_{K_1}(t), V_{K_1}(t^{-1})\}$, $\{V_{K_2}(t), V_{K_2}(t^{-1})\}$. If K_i , $i = 1, 2$, is amphichiral, we did not compare $\{V_{K_i}(t), V_{K_i}(t^{-1})\}$ and $\{V_{K_1}(t), V_{K_2}(t^{-1})\}$. When we found an equal pair, we examined their Kauffman and homfly polynomials. In addition, we compared $\{\nabla_{K_1}(z), \nabla_{K_2}(z)\}$ if K_1 and K_2 had the same genus.

4. COMPUTATIONAL RESULTS

Combining Facts 1 and 2 in the introduction and Table 1, we know all the pairs sharing the same polynomial invariants.

In Table 1 in the Supplement section at the end of this issue, the three numbers “ p, q, r ” represent the pair of the 2-bridge knots $\{S(p, q), S(p, r)\}$ in Schubert’s notation. If there is no mark, $\{S(p, \pm q), S(p, \pm r)\}$ share the same Q polynomial. If there is a mark “V” (resp. “P”, “F”, “PF”), the pair $\{S(p, q), S(p, r)\}$ shares the Jones (resp. homfly, Kauffman, homfly and Kauffman) polynomial. If $S(p, q)$ is not amphichiral, $\{S(p, -q), S(p, -r)\}$ is also such a pair. If there is a mark “C”, this pair shares the same Conway and Q polynomials. Note that we do not list the pair sharing only the same Conway polynomial. The mark “a” indicates that the knots are amphichiral. In this table, we have redundant information, for example, in 16 crossing knots, there are three pairs having the same Q polynomials: $\{S(429, 89), S(429, -353)\}$, $\{S(429, 89), S(429, -331)\}$, $\{S(429, -353), S(429, -331)\}$, which means the triple $\{S(429, 89), S(429, -353), S(429, -331)\}$ has the same Q polynomial. More complicated situations occur: Let $K_1 = S(1925, 569)$, $K_2 = S(1925, -1081)$, $K_3 = S(1925, 1229)$, which are 18 crossings. Then $P_{K_1} = P_{K_3}$, and $F_{K_2} = F_{K_3}$, and so $V_{K_1} = V_{K_2} = V_{K_3}$, $Q_{K_1} = Q_{K_2} = Q_{K_3}$, and $\nabla_{K_1} = \nabla_{K_3}$. Other equalities do not hold among them. Since they are not amphichiral, for the mirror images \bar{K}_i , $i = 1, 2, 3$, similar equalities hold.

In Table 2, for the n -crossing 2-bridge knots, we list the numbers of \mathcal{K}_n , \mathcal{K}_n^* and the pairs listed in Table 1. From this table, we obtain Figure 4, which presents what proportion of the 2-bridge knots fail to be determined by the homfly and Kauffman polynomials.

TABLE 2

n	# K_n	# K_n^*	No mark	V	P	F	PF	C	Total
10	45	85	2	1	0	0	0	0	3
11	91	182	0	0	0	0	0	1	1
12	176	341	7	3	0	0	0	0	10
13	352	704	2	4	0	2	0	6	14
14	693	1365	14	18	4	0	1	0	37
15	1387	2774	17	23	1	4	0	7	52
16	2752	5461	77	46	20	1	2	0	146
17	5504	11008	65	73	5	10	0	27	180
18	10965	21845	202	161	40	5	7	2	417
19	21931	43862	229	244	13	22	0	72	580
20	43776	87381	593	498	82	14	17	17	1221
21	87552	175104	669	960	24	46	0	186	1885
22	174933	349525	1607	1751	236	46	31	47	3718

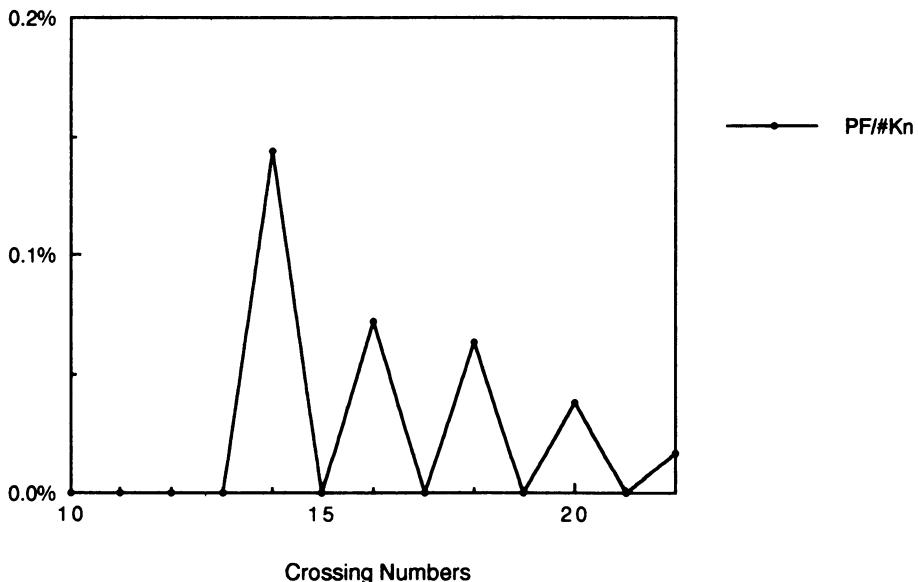


FIGURE 4

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Supplement to
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 THROUGH 22 CROSSINGS**

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Table 1

10 crossing	15 crossing	429 -89,331	693 -205,481	591 415,469	C
49 -15,27	V	429 -89,353	693 -191,481	595 -257,467	
51 11,23		429 331,353	693 -151,481	597 323,473	
57 -13,41		435 -191,341	693 -205,509	611 107,159	
11 crossing		441 -103,191	693 -191,509	623 447,489	C
213 163,169		451 -85,355	693 -151,509	623 -165,403	V
231 -107,191		455 -97,323	693 481,509	627 -119,347	
253 -113,205		459 107,143	693 -205,535	627 -115,457	
105 29,41	C	267 145,305	693 -80,257	693 -191,505	V
12 crossing		271 187,205	693 -151,335	693 -145,221	
279 -85,221	V	477 -200,389	693 481,535	P 646 203,227	C
63 -13,43		287 157,190	693 103,205	693 509,535	645 403,511
81 19,37	V	297 -161,235	V 483 -221,313	651 199,205	C
91 19,37		297 -67,241	483 -221,331	715 163,423	V
105 23,47		299 53,131	483 313,331	721 165,417	V
111 -25,77		301 79,135	483 -221,373	729 217,271	V
121 -49,49		319 -83,225	V 483 313,373	735 -223,533	V
143 -29,97		321 101,113	C 483 331,373	737 167,435	V
147 -43,83		323 -117,263	483 -113,379	747 -169,329	V
159 47,59		325 -69,231	489 -145,335	749 171,423	V
165 -49,119		327 91,103	C 492 107,277	V 749 -223,547	V
13 crossing		333 -71,271	493 -147,363	759 -205,587	V
341 -61,247		495 -91,271	V 765 173,227	V 703 -395,441	V
99 -17,71		343 -145,300	495 -223,400	777 -341,541	P 705 497,557
135 -37,107	V	345 -203,259	P 497 107,145	777 -203,541	711 -127,517
143 -27,105		351 -109,260	V 497 -151,341	779 -165,573	V
153 -83,121	V	357 253,281	C 497 317,383	781 571,615	735 -161,421
161 113,127	C	369 -113,283	V 507 -157,389	V 701 -355,549	735 -327,587
171 -97,131	V	385 117,173	V 517 -113,371	V 701 325,551	P 721 -221,563
189 -53,145	V	385 -71,281	V 525 -113,377	V 701 -179,577	V
203 -89,150	C	385 -163,303	C 529 -229,369	V 803 215,237	P 737 -169,437
213 65,77	V	399 121,163	V 532 321,413	V 819 187,229	V 737 -333,601
219 61,67	C	427 307,335	C 537 -125,295	833 -253,587	V 745 -303,417
231 167,181	F	429 131,155	C 539 -161,391	V 841 -231,523	V 747 -205,515
245 -103,137	V	441 -121,311	V 539 417,439	V 855 -181,629	V 749 -199,585
255 -143,163	F	459 -127,359	V 543 -123,235	V 867 511,613	V 753 577,589
14 crossing		473 -305,333	V 551 -243,327	V 875 -361,639	V 753 409,595
121 -23,65	V	473 193,365	V 551 -149,431	P 927 653,671	P 759 461,551
159 -35,71		475 -203,365	V 553 -149,435	P 931 285,614	P 759 431,581
161 -37,51		477 -263,373	V 553 -153,379	P 961 -203,559	V 772 583,613
165 -37,113		495 -203,277	F 555 -251,319	P 963 -407,677	P 781 -177,637
169 -31,125	V	495 -311,340	V 555 -119,397	V 979 280,619	P 783 -505,539
169 -53,129	V	495 -277,382	V 561 -163,313	P 981 -617,691	P 783 -217,611
207 43,61	V	495 203,383	V 561 -261,403	P 995 577,587	PF 799 143,211
209 -59,161	V	505 -203,323	F 561 -261,409	1005 -593,613	PF 801 -443,625
221 -41,163	V	513 -143,397	V 581 -123,435	1017 715,733	P 813 175,367
225 -47,103	V	517 117,139	V 593 -177,333	1083 -457,683	V 813 461,623
231 -41,125	V	531 -149,373	V 595 -269,439	P 1133 313,335	P 819 -457,635
231 53,137	V	551 -119,403	S 597 -139,461	17 crossing	V 819 -179,667
243 -55,100	V	649 -181,457	V 605 419,473	825 -233,647	
245 -111,130	V	745 -313,437	F 605 -219,411	V 827 -69,273	S 603 615
253 -47,139	V	755 -443,463	F 621 -341,485	P 839 -103,261	V 833 -689,650
253 -57,149	V	627 -341,487	P 621 -341,487	P 857 61,95	V 837 -539,577
259 50,143	V	16 crossing	623 141,279	309 -82,335	V 837 -233,649
259 -55,183	V	169 27,79	V 625 349,399	F 823 -133,335	V 841 -249,621
261 -61,113	V	213 47,95	629 -133,513	P 827 -235,339	V 851 -157,625
267 83,95	V	219 -49,149	637 -167,449	V 829 137,149	C 861 619,625
273 -85,197	V	245 39,59	638 -353,499	P 835 121,139	C 861 487,661
275 -73,213	V	255 -41,209	651 149,191	V 841 -239,349	V 867 665,671
275 65,83	V	275 -47,217	V 657 -367,509	P 845 -269,361	V 867 -239,679
287 61,121	V	287 -59,221	663 -193,487	P 845 89,109	C 869 -185,673
289 171,205	V	289 -101,239	V 665 -139,491	P 857 -91,295	V 873 -245,601
297 213,233	P	343 -99,195	V 671 485,529	P 857 -149,429	V 875 479,611
310 -11,125	V	351 73,109	V 675 -379,521	P 851 -301,407	V 889 233,377
315 -173,247	V	361 -75,267	V 675 487,523	P 853 -293,421	V 899 -437,623
321 205,223	V	363 -80,297	V 679 -201,201	P 853 -293,431	V 909 -477,641
327 -97,227	V	369 -77,169	V 679 -201,211	V 849 -177,421	V 913 -197,520
329 75,89	V	375 119,131	V 679 -187,471	P 853 393,435	C 915 209,251
329 -71,237	V	377 -99,307	V _a 679 -201,499	P 853 -149,439	V 915 197,257
333 -187,257	P	381 79,175	V 679 -187,499	V 857 -129,469	V 917 -543,717
351 253,271	P	381 -121,275	V 679 471,499	P 851 -247,457	C 921 199,253
361 -153,227	V	385 -87,263	V 693 151,191	P 853 -101,313	921 647,707
363 -131,265	V	405 91,181	V 693 151,205	P 853 -105,423	927 -283,725
505 293,313	PF _a	427 -127,293	V 693 191,205	P 853 -123,449	V 931 -389,523

SUPPLEMENT

Table 1 (continued)

Table 1 (continued)

Table 1 (continued)

Table 1 (continued) Table 1 (continued)

SUPPLEMENT

Table 1 (continued)

Table 1 (continued)

SUPPLEMENT

Table 1 (continued)

Table 1 (continued)

Table 1 (continued)

SUPPLEMENT

Table 1 (continued)

Table 1 (continued)

Table 1 (continued)