REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

13[01A60, 01A70, 68–03].—WILLIAM ASPRAY, *John von Neumann and the Origin of Modern Computing*, History of Computing, The MIT Press, Cambridge, MA, 1990, xvii+376 pp., $23\frac{1}{2}$ cm. Price $35.00.

By 1943, when he first became interested in computational mathematics, John von Neumann had established an international reputation based on original contributions to operator theory, logic, game theory, mathematical economics, and the mathematical foundations of quantum mechanics.

Biographical details prior to that time are presented in the opening chapter, providing the background for a fully documented, copiously annotated account of his abbreviated career in computer science. During that period, spanning slightly more than a decade, von Neumann organized the Electronic Computer Project at the Institute for Advanced Study and made lasting contributions to the theory, design, construction, and application of digital computers. Modern numerical analysis may be traced to his critical examination of methods for solving large linear systems, his development of Monte Carlo methods in collaboration with Stanislaw Ulam, as well as his research in linear programming methods and the numerical solution of partial differential equations arising in studies of fluid dynamics.

Also described in detail is his pioneering work with others in the application of computers as a scientific tool in diverse fields that include astrophysics, numerical meteorology, atomic and nuclear physics, biological information processing, and cybernetics.

A concluding chapter describes von Neumann's unabated activity after 1945 as a consultant to both government and industry in areas outside computing. This part of his career is conveniently summarized chronologically in a table that also includes a listing of his numerous awards, honors, and society memberships.

Appended to the text is an extensive set of explanatory and supplementary notes (83 pages) followed by a general bibliography of 346 items consisting of works cited in the text or used directly in its preparation, and finally a list of 190 writings authored or coauthored by von Neumann.

This meticulously researched, carefully written, and well illustrated book is indeed a fitting literary tribute to the enduring accomplishments of a remarkable man and his associates.

J. W. W.

14[65–01].—James L. Buchanan & Peter R. Turner, *Numerical Methods and Analysis*, McGraw-Hill, New York, 1992, xvi+751 pp., 24 cm. Price $44.95.

The text gives a careful explanation of how arithmetic operations are performed by computers (from pocket calculators to serial or parallel central processors). Here one of the novel features is the description of the CORDIC algorithms that are built into pocket calculators to do multiplication and division and to find the values of the elementary functions. Overwhelming is the amount of sound information in the densely filled pages of this book—where intuitively motivated numerical methods are algorithmically presented in TURBO PASCAL, mathematically analyzed, and then applied in illustrative examples that confirm the validity of the analysis and intuition. Readers will find each section to be relatively self-contained and supplied with many supplementary exercises and "projects".

"Ordinary" are the differential equations treated in Chs. 10 and 12 except for two projects suggested in the latter, one for heat conduction and the other for Poisson's equation in a rectangular region. Ultimately, an efficient code for solving a finite difference formulation of Poisson's equation is described for a hypothetical parallel computer in the last chapter. Good advice for performing a variety of basic tasks on a parallel system is also given there.

Here is a listing of the chapter headings in this classroom-tested opus that was skillfully prepared by experts and which will prove to be a wonderfully useful contribution to our literature:

	Preface – 3 pages
	General Introduction – 4 pages
Ch. 1	Computer Arithmetic and Errors – 44 pages
Ch. 2	Iterative Solution of Nonlinear Equations – 49 pages
Ch. 3	Approximate Evaluation of Elementary Functions – 34 pages
Ch. 4	Polynomial Interpolation – 44 pages
Ch. 5	Other Interpolation Functions – 45 pages
Ch. 6	Systems of Linear Equations – 45 pages
Ch. 7	Approximation of Functions – 80 pages
Ch. 8	Optimization – 73 pages
Ch. 9	Numerical Calculus – 72 pages
Ch. 10	Numerical Solution of Differential Equations – 106 pages
Ch. 11	The Eigenvalue Problem – 43 pages
Ch. 12	Boundary Value Problems for Differential Equations – 19 pages
Ch. 13	The Impact of Parallel Computers – 55 pages
App. A	Background Theorems in Real Analysis – 7 pages
App. B	Background in Linear Algebra – 8 pages
App. C	Answers to Selected Exercises – 14 pages
	Bibliography – 3 pages
	Index – 6 pages

E. I.

15[65-01].—GENE H. GOLUB & JAMES M. ORTEGA, *Scientific Computing and Differential Equations—An Introduction to Numerical Methods*, Academic Press, Boston, 1992, xi+337 pp., $23\frac{1}{2}$ cm. Price \$49.95.

This is a revision of "An Introduction to Numerical Methods for Differential Equations" by J. M. Ortega and W. G. Poole, Jr., Pitman Publ., 1981. The new version has the subtitle "An Introduction to Numerical Methods", which emphasizes that this text uses the numerical treatment of differential equations as a vehicle for discussing fundamental topics of numerical mathematics.

This concept goes back to the preceding version; it has been strengthened by the amplification of some sections dealing with subjects other than differential equations, notably linear equations and least squares. As in the earlier version, one feels that the authors are really more at ease when dealing with systems of equations, eigenvalues, etc. rather than with the numerical treatment of differential equations. While the level of exposition is elementary in both areas, the text is appreciably more concise and definitive in its formulations in the algebraic sections than it is in the analytic ones. For example, ill-conditioning is not discussed in the context of initial value problems, where it is first met, but only with linear systems of equations; the discussion of roundoff effects is much more vague in differential equations than in algebraic problems, etc. The fundamental distinction between the numerical solution behavior for initial value problems as $h \to 0$ on a fixed finite interval, and as the interval length grows for a fixed small h, remains as vague as the criteria upon which a step-size control may be based and what they effect. On the other hand, there are excellent introductory expositions on the least squares problem and orthogonal polynomials, on projection methods, and on the direct and iterative methods for solving large sparse systems coming from partial differential equations, and the like. The weaknesses go back to the original version; it is a pity that they have not been eliminated in the revision.

On the other hand, it is a tribute to the authors of the original text that the first chapter "The World of Scientific Computing" needed very little updating except in the hardware section and in the treatment of visualization and symbolic computation. Also, the refreshingly original basic concept of the text has remained a challenge over the past ten years. The many exercises have remained a great asset.

Altogether, I would recommend the book as a text for an introductory course in numerical analysis (on the right level), but not as an introduction to scientific computation and differential equations. Thus, the inclusion of the subtitle in a quotation of the title appears to be necessary to avoid any misleading impression.

H. J. S.

16[65-02].—WILL LIGHT (Editor), *Advances in Numerical Analysis*, Vol. I: *Nonlinear Partial Differential Equations and Dynamical Systems*, Clarendon Press, Oxford, 1991, x+275 pp., 24cm. Price \$52.00.

This book is the first of two planned volumes containing notes from a series of lectures delivered at the fourth Summer School in Numerical Analysis held at Lancaster University in 1990. It consists of six chapters, each corresponding to a short course of five lectures on a topic of current interest within numerical analysis of nonlinear partial differential equations and of dynamical systems. The intention of the editor appears to have been to create a semblance of one coherent treatise rather than a collection of independent contributions, and the various authors are not listed in the table of contents.

The first chapter, Finite Element Methods for Evolution Equations, by Lars B. Wahlbin, concerns mostly error analysis for linear parabolic equations and associated integro-differential equations with a memory term; it can be considered as introductory in the present context.

The second chapter, Finite Element Methods for Parabolic Free Boundary Problems, by R. H. Nochetto, treats the fixed-domain method for the Stefan problem. Approximations of the continuous problem by regularization and phase relaxation are described and used as motivation for the construction and analysis of discrete schemes based on finite elements in space and backward differencing in time. Much of the relevant material is in the form of exercises.

The third chapter, An Introduction to Spectral Methods for Partial Differential Equations, by A. Quarteroni, starts with reviewing basic properties of Fourier and Chebyshev expansions and corresponding discrete transforms, including fast ones. It proceeds with application to Galerkin and collocation methods for elliptic and parabolic boundary value problems, including the Navier-Stokes equations and ends with a section on domain decomposition methods.

The fourth chapter, Two topics in Nonlinear Stability, by J. M. Sanz-Serna, first defines stability for a linear problem as uniform well-posedness with respect to a discretization parameter, and then discusses generalizations to nonlinear problems. The second part is devoted to long-time behavior of discretized dynamical systems and of symplectic numerical integrators.

In the fifth chapter, Numerical Methods for Dynamical Systems, by Wolf-Jürgen Beyn, some of the topics covered concern numerical computation of invariant sets such as stationary points, periodic orbits and tori, the transition between these objects in parametrized systems, and the analysis of long-time behavior of time discrete trajectories.

The sixth and final chapter, The Theory and Numerics of Differential-Algebraic Equations, by Werner C. Rheinboldt, begins with three examples of applications of differential-algebraic equations and proceeds to discuss existence theory for implicit such equations, using techniques of modern differential geometry. The chapter closes with a section on numerical methods based on the notions developed.

The six chapters are somewhat different in style and degree of accessibility, with the contributions of Wahlbin, Sanz-Serna, and perhaps also Quarteroni, making for relatively easy reading. The topics of Nochetto's and Beyn's chapters, the two longest ones, are by their nature more difficult and require substantial efforts on the part of the reader, and Rheinboldt's presentation depends on a good familiarity with the language of algebraic geometry.

Together the contributions make for a very nice overview of important recent developments in numerical mathematics, and with the extensive lists of references the book should be a useful source of information.

V. T.

17[41–02, 65–02].—WILL LIGHT (Editor), *Advances in Numerical Analysis*, Vol. II: *Wavelets, Subdivision Algorithms, and Radial Basis Functions*, Clarendon Press, Oxford, 1992, viii+210 pp., 24 cm. Price $49.95.

This volume is devoted to three major topics in theoretical multivariate approximation. It contains three chapters, each reflecting a course of lectures at the Fourth Summer School in Numerical Analysis, University of Lancaster, 1990. The authors-lecturers and titles are as follows. 1. Charles K. Chui, "Wavelets and Spline Interpolation" (32 pages); 2. N. Dyn, "Subdivision Schemes in Computer-Aided Geometric Design" (63 pages); 3. M. J. D. Powell, "The Theory of Radial Basis Function Approximation in 1990" (106 pages). As explained in the preface, the exposition in the lectures (and the chapters) was to be "pitched at such a level that researchers and graduate students could both gain something useful from the courses". The level of expository writing is high, and the volume can be recommended for introducing the reader rapidly to the subjects addressed, and bringing her up to date in each.

E. W. C.

18[65–01, 65N38].—GOONG CHEN & JIANXIN ZHOU, *Boundary Element Methods*, Computational Mathematics and Applications, Academic Press, London, 1992, xx+646 pp., $23\frac{1}{2}$ cm. Price $87.00.

The object of the book is to present both the mathematical and the numerical background involved in the study of boundary integral methods. The authors have selected and synthetized, from many authors and books, all the tools which are necessary for a study of the subject.

The first three chapters contain a digest of the essential tools in functional analysis needed:

– Introduction to Sobolev spaces with the essential results (including some proofs)

– Sketch of the theory of distribution with many examples, including most of the usual finite part integrals.

In Chapter 4, they introduce pseudodifferential operators in $\mathbf{R}^n$. These are the classical elliptic ΨDOs considered as operators on Sobolev spaces. Then, in §4.4, they apply this theory to boundary integral operators (multiple-layer potentials for the Laplacian). Surprisingly, their definition of the symbol is wrong (formula (4.66), page 99), and they in fact compute the symbol of the image by the mapping (flattening $\partial\Omega$) of the Laplacian. This operator is different from the Laplacian, but not too much, so that their result is almost correct and the error does not affect the rest of the book. In §4.5, they present the Calderón operator, in a quite general and complicated manner. But the basic results are clearly obtained. They end this chapter with a classical but neat presentation of the Fredholm theory in Sobolev spaces, with some applications to boundary integral equations for the Laplacian.

Chapter 5 is devoted to a presentation of finite element theory with all the classics:

– Variational formulation and Lax-Milgram theorem

– Inf. Sup. conditions

- The main families of finite elements
- Inverse inequality and Aubin-Nitsche theorem.

It has the merit of being short and comprehensible.

The rest of the book is devoted to applications of the previously introduced tools. In Chapter 6 the Laplace equation is considered and the authors present the essentials of Giraud's theory, i.e., the $C^{0,\alpha}$ theory of the potentials and jump properties associated with the Laplacian, and they deduce all the classical limit behavior at the boundary of these potentials. Using then the pseudodifferential operator results in the case of data in some Sobolev spaces, they extend these properties. They also give the coerciveness property of the simple-layer potential (with a wrong historical reference). We also must mention that the finite parts used by the authors are incorrect to treat the double-layer potentials (page 279 and following). There follow some numerical examples, quite elementary, but interesting from the pedagogical point of view. Chapter 7 is devoted to the Helmholtz equation. It contains a nice presentation of most aspects of the problem, including the radiation condition, the problem of interior eigenvalues and also some nice numerical experiments (in color). Chapter 8 concerns the plate problem and is rather confusing, although the main ideas are quite clear. The presentation of the triple or quadruple layer is probably original but only sketched. Chapter 9 contains a classical treatment of elastostatics and would be simplified by the use of a quotient space. Chapter 10 contains some aspects of error estimates with emphasis on approximation by splines (and collocation).

In conclusion, despite some weaknesses, the book is well written and quite clear. Most of the material is largely classical and already contained in some text books. But the selection of topics is good in general, and it is probably one of the first self-contained books on the subject of boundary integral equations (except [1]). This is its main interest that will make it very useful for Ph.D. students working on this subject.

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1. R. Dautray and J.-L. Lions, *Analyse mathématique et calcul numérique pour les sciences et les techniques*, tome 3, Masson, Paris, 1985.

19[65–06, 65Lxx, 93–06].—EDWARD J. HAUG & RODERIC C. DEYO (Editors), *Real-Time Integration Methods for Mechanical System Simulation*, NATO ASI Series, Series F: Computer and Systems Sciences, Vol. 69, Springer, Berlin, 1991, viii+352 pp., 25 cm. Price \$79.00.

Among the many published proceedings, this one from an August 1989 NATO Advanced Research Workshop can be strongly recommended. The authors of the 18 contributions work in mechanical engineering and numerical mathematics. All come from industries, universities, and research institutes. The talks deal with the numerical performance of multibody system formulations for specific examples and in general. The resulting mathematical model is a differential-algebraic equation (DAE) or, in the case of state-space coordinates, an ordinary differential equation (ODE). In applications, a DAE can be generated automatically on a computer and serves as a possible basis for a CAD/CAE package. But one has to pay a price for this automatic modelling of a multibody system. The DAE includes redundant information; this causes new theoretical and numerical problems. A desired set of state-space coordinates may require analytical work and then results in an ODE whose theoretical and numerical problems are well understood. On the other hand, it is difficult to incorporate tricky analytical solution techniques into a software package. These few remarks on DAEs, ODEs, and mechanical systems may serve as a backdrop for the contributions in this book, which are grouped in three parts.

The first part consists of seven reports, which are introductory in character. Multibody system formalisms are introduced and applied to examples in articles by C. Deyo (vehicle suspension); H. Frisch (general problem description); R. Beck (army vehicles); R. Schwertassek, W. Rulka (wheel-rail systems, vehicles, robots). The theoretical and numerical properties of DAEs are discussed by C. W. Gear (index definition) and L. Petzold (DAE solver DASSL). A coordinate partitioning method for the numerical solution of multibody systems is presented by E. Haug and J. Yen.

The second part includes six contributions mainly dealing with the question of how to overcome the index problem of DAEs. Mechanical systems with constraints on the acceleration level are of index 1; constraints on the velocity level lead to index-2, and constraints on the position level lead to index-3 DAEs. Today the solution of index-1 DAEs is well understood, whereas DAE with index greater than 1 are a topic of current research. C. Führer and B. Leimkuhler study the overdetermined approach and show the equivalence to a state-space formulation. Ostermeyer discusses the Baumgarte approach as a control-theoretical tool, see also D. Bae and S. Yang. A different view of DAEs as differential equations on manifolds is given by F. Potra and W. Rheinboldt. Parallelization techniques like waveform relaxation and multirate methods are discussed by C. W. Gear and by W. Bruijs et al.

Different DAE applications are summarized in the five contributions of the third part. J. G. de Jalon et al. present several interesting mechanical systems, developing a special multibody system approach. S. Sparschuh and P. Hagedorn discuss a discretization of the Gauss principle. Control theory and applications are discussed by W. Cotsaftis and C. Vibet. Different numerical discretization schemes—among them the BDFs—for multibody formalisms are studied by J. Meijaard and by M. Steigerwald.

In summary, these proceedings present a broad spectrum of interesting current research in multibody system dynamics and in DAE techniques. The book is a must for libraries and researchers in this field. For nonspecialists it presents a valuable introduction to modern CAD/CAE tools based on numerical techniques and engineering models, with interesting applications.

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20[65-06, 65N30].—J. R. WHITEMAN (Editor), *The Mathematics of Finite Elements and Applications.* VII, *MAFELAP* 1990, Academic Press, London, 1991, xvi+637 pp., $23\frac{1}{2}$ cm. Price \$65.00.

This volume contains the text of the invited and contributed papers (44) and abstracts of lectures in parallel and poster sessions (29) from the seventh MAFELAP conference, held at Brunel University in April 1990. As usual, they range over a very broad span of topics in finite element analysis.

L. B. W.

21[65-06].—GARY COHEN, LAURENCE HALPERN & PATRICK JOLY (Editors), *Mathematical and Numerical Aspects of Wave Propagation Phenomena*, SIAM Proceedings Series, SIAM, Philadelphia, PA, 1991, xiv+794 pp., $25\frac{1}{2}$ cm. Price: Softcover \$68.00.

This volume is the proceedings of the First International Conference on Mathematical and Numerical Aspects of Wave Propagation Phenomena, held in Strasbourg, April 23–26, 1991. It consists of eleven parts containing seventy-two papers (and, a twelfth part of twenty-three poster sessions). Part 1, Numerical Methods, contains sixteen papers. The other parts are "Modelling", "Boundary Conditions and Control", "Scattering Problems", "Surface Waves/Hydrodynamics", "Inverse Scattering Problems", "Nonlinear Waves", "Wave Propagation in Random Media", "Resonances, Guide Waves and Layered Media", "Homogenization/Asymptotic Analysis". Many papers in those sections contain simulations and some have a strong numerical methods content.

L. B. W.

22[42C15, 41A30, 78A40].—CHARLES K. CHUI, *An Introduction to Wavelets*, Wavelet Analysis and Its Applications, Vol. 1, Academic Press, Boston, 1992, x+264 pp., $23\frac{1}{2}$ cm. Price \$49.95.

The appearance of this book is certainly welcome, as it is probably the first textbook in English on the subject of wavelets. It also inaugurates a series called "Wavelet Analysis and Its Applications" undertaken by Academic Press and edited by Charles Chui. The series will contain monographs and edited collections of papers. The chapter titles of the book under review are: 1. An Overview, 2. Fourier Analysis, 3. Wavelet Transforms and Time-Frequency Analysis, 4. Cardinal Spline Analysis, 5. Scaling Functions and Wavelets, 6. Cardinal Spline-Wavelets, and 7. Orthogonal Wavelets and Wavelet Packets. There is a five-page section of historical notes and a five-page list of references, most of which date from 1984 onward. Chui's book would furnish an excellent backbone for a course on wavelets; it is nicely organized and contains proofs of all results—exactly as one would expect from this very active and influential mathematician.

E. W. C.

23[65-02, 26C05, 33C45, 42C05, 65D99].—CLAUDE BREZINSKI, *Biorthogonality and Its Applications to Numerical Analysis*, Monographs and Textbooks in Pure and Applied Mathematics, Vol. 155, Dekker, New York, 1992, viii+166 pp., $23\frac{1}{2}$ cm. Price \$85.00.

The last decade has witnessed a quiet revolution in the theory and application of orthogonal polynomials. Of course, numerical analysts and practitioners of scientific computing have been always aware of the role of orthogonal polynomials in quadrature methods, spectral algorithms, numerical algebra, approximation theory, etc. Recent work, however, went a long way not just to emphasize the centrality of orthogonal polynomials to computation but to highlight a wide range of further applications—from quantum groups to dynamical systems, from coding theory to spectral properties of the Schrödinger equation, from group representation to signal processing... [7].

An important part of the unfolding orthogonal scene are 'exotic' concepts of orthogonality, e.g., convolution orthogonality [1] and orthogonality with respect to a Sobolev inner product [6]. A place of pride on this list belongs to biorthogonal functions, the subject of the book under review. And who can be a more natural expositor of this subject area than Professor Brezinski, one of the leading workers on both theory and applications of biorthogonality?!

There are several ways of introducing biorthogonal functions and the book follows the original framework of Davis [2]. Let E be an infinite-dimensional vector space and let $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots \in E$ be linearly independent. Moreover, suppose that $L_0, L_1, L_2, \ldots$ are linear functionals acting on E (i.e., elements of the dual space E^*) and that

$$\det \begin{bmatrix} L_0(\mathbf{x}_0) & L_1(\mathbf{x}_0) & \cdots & L_n(\mathbf{x}_0) \\ L_0(\mathbf{x}_1) & L_1(\mathbf{x}_1) & \cdots & L_n(\mathbf{x}_1) \\ \vdots & \vdots & & \vdots \\ L_0(\mathbf{x}_n) & L_1(\mathbf{x}_n) & \cdots & L_n(\mathbf{x}_n) \end{bmatrix} \neq 0$$

for all $n = 0, 1, 2, \ldots$. Then there exist unique linear combinations

$$L_n^* = \sum_{j=0}^{n} a_{n,j} L_j, \quad a_{n,n} \neq 0, \qquad \mathbf{x}_n^* = \sum_{j=0}^{n} b_{n,j} \mathbf{x}_j, \qquad n = 0, 1, 2, \ldots,$$

such that $L_n^*(\mathbf{x}_m^*) = \delta_{m,n}$. The set $\{L_n^*, \mathbf{x}_n^*\}_{n=0}^{\infty}$ is called a *biorthogonal family.*

Other definitions of biorthogonality require somewhat stricter frameworks. Nevertheless, they are highly illuminating and the loss of generality does not interfere with realistic applications. Thus, if E is a Hilbert space, then by the Riesz equivalence theorem biorthogonality can be expressed in terms of inner products. This is somewhat more 'symmetric', since the two sequences required for biorthogonality belong to the same space. Furthermore, if the inner product is selfadjoint, then the Daniel theory of integration implies that biorthogonality can be expressed in terms of Stieltjes-Lebesgue measures (or, for those at home with generalized functions, weight functions). The latter framework is particularly useful and transparent in the case of biorthogonal polynomials—more about it in the sequel.

The first part of the book is devoted to a formal exposition of biorthogonal functions and their theoretical features. It brings together a great deal of results, many quite recent, that have been scattered throughout the scientific literature. They are bound together with a generous measure of mathematical mortar, the cracks and gaps being filled by original and unpublished research. The formal functional-analytic approach does not make for an easy reading, but the extra effort required to master the contents and to follow its technical minutiae is worthwhile. It is, however, the sentiment of this reviewer that this part is needlessly formal and uncompromising in its attitude to the reader. Very seldom are we told the purpose of this or that construct at the point of embarkation. Instead, the exposition, as elsewhere in the book, is a Bourbaquiste progression of definition → proposition → theorem. The material of the book is *intrinisically* difficult and calls for mathematical maturity and ability to cope with subtle technicalities on the reader's part—why make it even more straightlaced and formal?

The surfeit of formalism occasionally hides gaps and ambiguities. Thus (pages 25–26, verbatim):

"3.5 The method of moments

This method, studied by Vorobyev [186] in a Hilbert space, is a particular case of Galerkin's method. We shall now extend it to an arbitrary vector space E and its dual E^*.

The method of moments consists in constructing a linear operator A_n on E_{n-1} such that

$$\begin{aligned} x_1 &= A_n x_0 \\ x_2 &= A_n x_1 \\ &\cdots \\ x_{n-1} &= A_n x_{n-2} \\ I_{n-1}(x_n) &= A_n x_{n-1} \end{aligned}$$

or

$$\begin{aligned} x_k &= A_n^k x_0 \qquad k = 0, \ldots, n-1 \\ I_{n-1}(x_n) &= A_n^n x_0 . \text{"} \end{aligned}$$

We are told neither what is E_{n-1} (as distinct from E) nor whether the definition should be valid for just one $x_0 \in E_{n-1}$ or for all $x_0 \in E_{n-1}$. The interpolation operator I_{n-1} has been already defined in §3.2 in a different formalism—the reader should be able ultimately to work out its relevance in the present setting, but only with a wholly unnecessary extra effort. Most importantly, what is the purpose of the method of moments? Which problems is it supposed to solve? What are the advantages in studying it in arbitrary vector spaces?

To be fair, formalism has its rewards and, in Professor Brezinski's hands, it frequently becomes a powerful tool. An example, dear to this reviewer's heart, are Christoffel-Darboux identities for biorthogonal functions. Their derivation in [4] for the special case of biorthogonal polynomials required much effort (and, to be frank, much tedium), whereas the book presents a far-reaching generalization, based on its formalism—and does it, comprehensively, in a short page. Bravo!

In other instances, the quest for generality obscures important features of special cases. Thus, biorthogonal polynomials have been defined in [3] as follows: a nonzero nth-degree polynomial $p_n(\cdot; \mu_1, \mu_2, \dots, \mu_n)$ is biorthogonal with respect to the parametrized Borel measure $d\varphi(x, \mu)$ if

$$\int_{-\infty}^{\infty} p_n(x; \mu_1, \mu_2, \dots, \mu_n)\, d\varphi(x, \mu_\ell) = 0, \qquad \ell = 1, 2, \dots, n.$$

This definition can be made to fit into the straightjacket of this book's formalism by defining the functionals

$$L_k^* f := \int_{-\infty}^{\infty} f(x) d\varphi(x, \mu_{k+1}), \qquad k = 0, 1, \dots,$$

but there is a price to pay. Biorthogonal polynomials from [3] depend on their parameters $\mu_1, \dots, \mu_n$ in a continuous (indeed, differentiable) manner. This is absolutely crucial to their application to trace loci of zeros of polynomial transformations [5].

The remainder of the book presents an exposition of a considerable number of applications of biorthogonality. The list is impressive: the Lanczos method, biconjugate gradients, rational approximation theory, acceleration of convergence, design of multistep methods for ordinary differential equations and least squares calculations. Claude Brezinski speaks with great authority on all these and has been a driving force in the implementation of biorthogonal techniques. Thus, I approached this part of the book with great anticipation but, alas, found the paucity of explanation and motivation a real stumbling block. Thus, on page 33 we are treated to the only explanation of what the Lanczos method and biconjugate gradients are all about:

> "In a Hilbert space it is well known that the method of moments gives rise to Lanczos' method and then to the conjugate and biconjugate gradient methods, see [17, pp. 79–91, 186–189]."

Well, an educated numerical analyst should have heard of the Lanczos method (and is unlikely to confuse it with the Lanczos τ method), although a brief reminder would have been welcome. But how many know of biconjugate gradients? Readers with plenty of commitment, motivation and spare time (to say nothing of a well-equipped library) may always consult the references—but what of the remaining 99%?

This is a book evidently written in a hurry. It is based on deep knowledge and scholarship and will be indispensable as a source for the small band of workers in the subject. I am, however, sceptical of its potential to popularize the important concept of biorthogonality in the broader numerical community.

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1. W. A. Al-Salam and M. E. H. Ismail, *Polynomials orthogonal with respect to discrete convolution*, J. Math. Anal. Appl. **55** (1976), 125–139.

2. P. J. Davis, *Interpolation and Approximation*, Blaisdell, New York 1963.

3. A. Iserles and S. P. Nørsett, *On the theory of biorthogonal polynomials*, Trans. Amer. Math. Soc. **306** (1988), 455–474.

4. ——, *Christoffel-Darboux-type formulae and a recurrence for biorthogonal polynomials*, Constructive Approx. **5** (1989), 437–453.

5. A. Iserles, S. P. Nørsett, and E. B. Saff, *On transformations and zeros of polynomials*, Rocky Mountain J. Math. **21** (1991), 331–357.

6. F. Marcellán, M. Alfaro, and M. L. Rezola, *Orthogonal polynomials on Sobolev spaces: old and new directions*, Universidad de Zaragoza Tech. Rep. **1** (1992).

7. P. Nevai (ed.), *Orthogonal Polynomials: Theory and Practice*, Kluwer Academic Publishers, Dordrecht, 1990.

24[40–01, 40–04, 65B10].—CLAUDE BREZINSKI & MICHELA REDIVO ZAGLIA, *Extrapolation Methods: Theory and Practice*, Studies in Computational Mathematics, Vol. 2, North-Holland, Amsterdam, 1991, x+464 pp. (includes floppy disk), $24\frac{1}{2}$ cm. Price \$115.50/Dfl. 225.00.

In the applied sciences one often faces the task of determining a numerical value for the limit of a slowly convergent sequence. In many situations the sequence is divergent, yet there is a commanding physical reason to attach a meaning—in the sense of limit—to the sequence. Techniques for summing slowly convergent, or divergent, sequences go by the generic name of *summation methods*. The idea is to transform the given sequence S_n into a sequence $\overline{S}_n$ by some kind of formula, $\overline{S}_n = F_n\{S_0, S_1, \dots, S_{k(n)}\}$, $n = 0, 1, 2, \dots,$ so that $\overline{S}_n$ converges to the same limit, but more rapidly.

In such an undertaking the numerical analyst has to address several issues, partly philosophical in nature:

1) For a given sequence or class of sequences, which is the best technique to use?

2) What assurance does one have that the approximate limit will be close—arbitrarily close—to the true limit?

3) If the sequence is divergent, how can one know that the so-called limit calculated will reflect what the physical situation dictates?

We can easily dispose of the last dilemma. There can be *no* general assurance that the limit calculated is the "correct" one. In his book [3] on infinite series, Knopp gives an example to illustrate that several heuristically plausible "limits" can be assigned to a divergent sequence. Although textbook examples may be so concocted, reality seems gentler to the numerical analyst: it is a rule of thumb that in real-life situations one either gets no limit at all or the correct limit.

At a conference in January 1992 in Tenerife, E. J. Weniger presented some remarkable examples. The sequences in question were the partial sums, strongly divergent, of perturbation expansions for the ground state energies of the quartic, sextic, and octic anharmonic oscillators. The sequences posed challenging test problems for available summation methods, since their terms diverged, respectively, like $n!/n^{1/2}$, $(2n)!/n^{1/2}$, $(3n)!/n^{1/2}$. A favorite method—the Levin transformation—could not sum any of these sequences, and the failure was not an artifact of numerical instability or round-off error—a common pitfall of summation methods. Weniger performed the computations in Maple in 1000-digit precision; the failure of the Levin method was genuine. Another transformation *did* sum the sequences. It is interesting that in all the cases Weniger studied, the summation methods chosen either did not produce a convergent sequence or summed the sequence to its physically correct limit. Never did the methods deliver up a spuriously convergent sequence.

We can address the first and second issues in a more illuminating and productive way, and the present book presents an exhaustive survey of our current state of knowledge. One difficulty with working in this field is that crucial results are scattered throughout a voluminous technical literature, in technical reports, conference proceedings, published journals. Some remarkable theoretical and practical tools have emerged in the last decade. This book provides an account of them all. The senior author, Claude Brezinski, is a—perhaps *the*—leading authority on the subject, and the presentation reflects his expertise, experience, and organizational skills. One hat he wears is that of a mathematical historian, and the book is informed with a strong and compelling sense of the cultural primacy of numerical analysis, a subject that has fascinated the greatest mathematicians. This volume is a momentous contribution to the literature on computational mathematics.

Among very recent theoretical results are theorems describing those classes of convergent sequences that are accelerable, i.e., for which there is a summation method that will transform each sequence in the class into a more rapidly convergent sequence with the same limit. I asked in my book [6] on the subject—written in 1981 and, alas, quite out of date now—whether there was a universal method that would accelerate the convergence of all complex convergent sequences. The French school—consisting mostly of Claude and his present and past students—took a penetrating look at this question. The result was probably the most remarkable mathematics ever done in this area. The book quotes a beautiful theorem due to Jean Paul Delahaye (1988). First, a little notation. Let E be a metric space. For $M \subset E$ denote by M^* the set of accumulation points of M. Let $\mathbb{S}(E)$ be the class of convergent sequences in E with the property that for every sequence $\{S_n\} \subset \mathbb{S}(E)$ and every n_0, $S_n \neq \lim_{n\to\infty} S_n$ for some $n > n_0$.

Theorem. *A necessary and sufficient condition for* $\mathbb{S}(E)$ *to be accelerable is that* $(E^*)^* = \varnothing$.

This means, essentially, that if the class of its convergent sequences is to be accelerable, the metric space must be small, *very small*. The combined ranges of the sequences cannot even contain a perfect subset. The theorem is *general*. The older, classical summation methods are linear and generally offer only limited improvement in convergence. In the past 50 years or so, many highly effective nonlinear methods have appeared. The form of the method is irrelevant, however: the theorem makes no assumptions about the linearity of the sequence transformation used for the acceleration.

Researchers have discovered other striking facts. The union of two accelerable classes may be nonaccelerable. Also, it was a disappointment to those working in the area to find that the class of real monotone sequences is not accelerable. However, the class of complex linearly convergent sequences, i.e., convergent sequences with the property

$$\lim_{n\to\infty} \frac{S_{n+1} - S}{S_n - S} = \lambda, \qquad 0 < |\lambda| < 1, \ \lim_{n\to\infty} S_n = S,$$

is accelerable.

Current theoretical research has centered on defining more accurately the frontier between accelerability and nonacceleraabiltiy. What is the smallest non-accelerable subclass of convergent sequences? the largest accelerable subclass? The impossibility of finding a universal method has caused some authors to toy with the definitions of acceleration and transformation. Recent work in this area, due to researchers such as Wang, Jacobsen, and Germain-Bonne, has been fruitful.

Despite these intriguing theoretical observations, the book really aims for the problem reader, one with no specialized knowledge of summation methods, but with many, many obstinate sequences to sum. I have heard that scientists working in neutron diffusion problems encounter sequences consisting of vectors with tens of thousands of components. The attempt to find a numerical value of the limits of such a sequence will surely make a pragmatist of anyone.

The book starts out at square 1. Appropriately, the first section of the first chapter is entitled, *First steps.* To orient the reader, the authors at the outset quote two methods, one that of the arithmetic means (linear), and the other the Aitken Δ^2 method (nonlinear):

$$\overline{S}_n = \frac{S_n + S_{n+1}}{2}, \qquad n = 0, 1, 2, \ldots,$$

$$\overline{S}_n = \frac{S_n S_{n+2} - S_{n+1}^2}{S_{n+2} - 2S_{n+1} + S_n}, \qquad n = 0, 1, 2, \ldots.$$

These methods are old, and familiar to every numerical analyst, but they serve as paradigms for more modern approaches. The authors point out that the second process, rediscovered by Aitken, was known to the Japanese mathematician Seki Kowa (1642?–1708), who used it to accelerate the computation of π by the method of inscribed polygons.

Linear methods are the grandmothers of the subject: predictable, but tepid. Nonlinear methods are the bikers: robust, but erratic. Nonlinear methods are seldom regular, but they can sum vigorously divergent sequences.

Attempts to generalize and extend the two methods above have given rise to the contemporary literature on the subject. For instance, how can one extend the Aitken Δ^2 method to spaces where no inverses exist, for instance, topological vector spaces? Surprisingly, people have discovered such extensions. The methods, which employ the dual space, are ingenious.

We get in this chapter the basic definitions of *regularity* of an extrapolation algorithm (what I have been calling a summation method): $\overline{S}_n$ must converge to the same limit as S_n for any convergent S_n, of *accelativeness*—a bad word; is there a better one?—, of translativity, homogeneity, quasilinearity of algorithms. The *kernel* of a method is that subclass of sequences mapped identically into a constant sequence, the constant being the limit when the sequence is convergent. Constructing the kernel of an algorithm is obviously an important thing to do, since the kernel gives some sense of the types of sequences for which the algorithm will be effective. For the first process above, the kernel is the set of sequences of the form $S_n = S + a(-1)^n$. For the second it is the set of sequences of the form $S_n = S + a\lambda^n$.

In this chapter the authors establish a few essential principles: (i) One can express many noteworthy sequence transformations, for instance, the Shanks transformation and the Wynn ε-algorithm, based on it, as ratios of determinants involving members of the original sequence. (ii) The numerical stability of a method is often a consequence of how one formulates the method. (iii) A knowledge of the asymptotic properties of real sequences, properties sometimes obscure and often new, is essential to an understanding of the success of extrapolation methods.

The authors also discuss the most popular scalar extrapolation methods in Chapter 1. Many algorithms go by the name of the mathematician who discovered, more often than not, *rediscovered*, them: the Shanks transformation, the Levin transformation, the Germain-Bonne transformation, the Thiele extrapolation process. Let me discuss briefly the Shanks transformation. Anyone not familiar with the subject will find this algorithm, and the ε-algorithm based on it, very mysterious. Assume that the original sequence S_n converges to its limit S in the following fashion:

$$S_n \sim S + c_1\lambda_1^n + c_2\lambda_2^n + c_3\lambda_3^n + \cdots + c_k\lambda_k^n .$$

In the parlance of electrical engineering, one assumes the sequence is its limit plus a finite number of exponential transients. If one replaces the "$\sim$" by "$=$" above, one may solve for S by eliminating the exponentials. This produces an approximation to S, which in reality is a transformed sequence $\overline{S}_n$:

$$e_k(S_n) = \overline{S}_n = \frac{\begin{vmatrix} S_n & \cdots & S_{n+k} \\ \Delta S_n & \cdots & \Delta S_{n+k} \\ \cdots & & \cdots \\ \Delta S_{n+k-1} & \cdots & \Delta S_{n+2k-1} \end{vmatrix}}{\begin{vmatrix} 1 & \cdots & 1 \\ \Delta S_n & \cdots & \Delta S_{n+k} \\ \cdots & & \cdots \\ \Delta S_{n+k-1} & \cdots & \Delta S_{n+2k-1} \end{vmatrix}} . \tag{1}$$

A sensible tactic is to fix k, the number of transients, then allow n to get large and hope that $\overline{S}_n$ will converge more rapidly than S_n. For instance, $k = 1$ gives the Aitken Δ^2 method.[1] But what if the number of transients *and* n both get larger simultaneously? This speculation, a truly innovative one, caused a conceptual revolution in the subject. One generates a (n, k) table, in which one can travel downwards along an *arbitrary path*, hoping to select the path that will promote the most rapid convergence. But, of course, the determinants involved rapidly become unwieldy. Is there a better way of formulating the algorithm? There is, and it was discovered in 1956 by Peter Wynn [7].[2]

Define the sequence $\varepsilon_k^{(n)}$, $k, n = 0, 1, 2, \ldots$, by the following algorithm, sometimes called a *tableau*,

$$\varepsilon_{k+1}^{(n)} = \varepsilon_{k-1}^{(n+1)} + \frac{1}{\varepsilon_k^{(n+1)} - \varepsilon_k^{(n)}}, \qquad k, n = 0, 1, 2, \ldots, \tag{2}$$

with initial conditions $\varepsilon_{-1}^{(n)} = 0$, $\varepsilon_0^{(n)} = S_n$, $n = 0, 1, 2, \ldots$.

[1]More history: The case $k = 2$ of the algorithm was used by James Clerk Maxwell, 1892. Schmidt [4] discovered the general case in 1941, Shanks only later [5, p. 39], in 1949.

[2]Any honest reader will admit to wishing that he or she had discovered this algorithm first. I do.

One can show that

$$\varepsilon_{2k}^{(n)} = e_k(S_n). \tag{3}$$

How show? Alas, there seems to be no simple way. The proof Wynn gave depended on complicated determinant transformations known as Schweinsian identities.[3] More annoying is that one has to know that one wants to derive (3) to prove it. This is the Catch-22 of the mathematical enterprise. If only God gave us the formulas and we returned the proofs. Or vice versa. I have heard that Wynn based his supposition of (3) on a misreading of (1).

The ε-algorithm is dreadfully nonlinear, and thus, one might suppose, resistant to analysis. Yet, we now know a lot about it. A necessary and sufficient condition that a sequence S_n lie in the kernel of the ε-algorithm is that there exist $a_0, a_1, a_2, \ldots, a_k$ with $a_k \neq 0$ and $a_0 + a_1 + a_2 + a_k \neq 0$ such that

$$a_0(S_n - S) + a_1(S_{n+1} - S) + \cdots + a_k(S_{n+k} - S) = 0. \tag{4}$$

This beautiful result is due to Brezinski and Crouzeix (1970).

The reviewer and his coworkers have generalized the algorithm to abstract sequence spaces, and we can now completely characterize its effect on many complex sequences, at least for certain paths in the tableau. We have, for instance, the following result:

Let

$$S_n \sim S + \sum_{i=1}^{\infty} a_i (n+b)^{-i}, \qquad n \to \infty.$$

Then for k fixed and $n \to \infty$,

$$\varepsilon_{2k}^{(n)} \sim S + \frac{a_1}{(k+1)(n+b)}.$$

The authors display this result and many other tidbits in their Theorem 2.19.

The effect the ε-algorithm has on certain sequences is dramatic. For instance, it will sum the partial sums of the wildly divergent series

$$\sum_{n=0}^{\infty} (-1)^n n!,$$

in the sense that

$$\lim_{k\to\infty} \varepsilon_{2k}^{(0)} = .5963473623 = S.$$

S is the "correct" value, i.e.,

$$S = \int_0^{\infty} \frac{e^{-t}}{1+t}\, dt.$$

(Just expand $(1+t)^{-1}$ in its Taylor series and formally integrate term-by-term.)

Unfortunately, the algorithm is unstable—for instance, it must be expected that if the algorithm is to be effective, then when k is large, $\varepsilon_k^{(n)}$ and $\varepsilon_k^{(n+1)}$ will be close to each other during the computations. That is precisely what will produce a small denominator in the algorithm, and it tends to happen when S_n approaches its limit monotonically. Further, the algorithm is not

[3]There is a proof utilizing the theory of S-fractions [1], but it is not for the faint of heart (nor the unwary; there are crippling errors in text).

very flexible: it cannot accommodate itself to the shape of the sequence under consideration. Despite its elegance and theoretical importance—in the theory of Padé approximants, for instance—it is not today a significant numerical tool.

Numerical analysts often, but not universally, acknowledge that the two most effective algorithms are the Levin and the Θ-algorithm, the latter due to Brezinski. Problem readers needing a quick fix should read the material in Chapter 2, which introduces these algorithms and describes some of their properties. I caution the reader that the algorithm should depend on the application. If you want to accelerate the computation of definite integrals, think about the G-transformation. For monotone sequences, think about the Levin transformation. There is no universal algorithm. Statistical sequences, or sequences whose error sequences have erratic signs, will defeat any method.[4]

In applying any flexible extrapolation algorithm, it is very helpful to have an idea of the shape of the error sequence. The algorithm can then be tailored to that sequence. For many sequences, one can construct an asymptotic expansion of the error that will permit a more efficient use of an algorithm. Also, it may be possible to extract a subsequence from the given sequence and to accelerate the convergence of that subsequence rather than trying to tackle the original sequence. This is useful when the errors of the sequence have erratic behavior. The authors discuss these matters in Chapter 3. From their account of the large number of special devices one can employ, it is clear that the effective use of extrapolation algorithms is still much more of an art than a science.

As mentioned previously, vector sequences arise often in physics and engineering, usually in the numerical solution of partial differential equations by finite differences or the method of finite elements, and in matrix eigenvalue computations. Of course, one can simply use an extrapolation algorithm on each component of the vector sequence. Such an approach is neither effective nor intellectually satisfying. The theory of vector extrapolation is more sophisticated and has a connection with projection methods, which play an esteemed role in numerical analysis. I want to say some words about the vector ε-algorithm, since it has an intriguing theory and shows just how far ingenuity can take you in this business. If one attempts to apply the tableau (2) to vector sequences, the necessity of inverting vectors proves an impediment. Not to worry. Why not take as the inverse of a vector y the vector

$$y^{-1} = \frac{y}{(y\,,\,y)}\,,$$

which gives the right thing when y is a scalar? The algorithm so defined is the *vector ε-algorithm.* McLeod and Graves-Morris studied the kernel of the vector ε-algorithm and discovered that the condition (4) is *sufficient* for a vector sequence to belong to the kernel. The characterization of necessity is much deeper, and the authors devote some space to it. No one knows whether determinant expressions similar to (1)–(3) hold for the algorithm. Another strategy is to *start* with a determinant representation, then formulate a recursive algorithm. The result is called the *topological ε-algorithm.* The problem is that

[4]In my book I give some methods due to myself and Bob Higgins for accelerating the convergence of statistical sequences. But now I see these methods as mired in meta-mathematical dubieties. What does it mean to accelerate the convergence of a sequence which only converges almost surely? At one time I thought I knew. Now I'm not so certain.

the formula (1) makes sense only when one is working in a field. The way of getting around this is really clever. Let y be an arbitrary vector. Interpret the determinant (1) as

$$e_k(S_n) = \frac{\begin{vmatrix} S_n & \cdots & S_{n+k} \\ \langle y, \Delta S_n\rangle & \cdots & \langle y, \Delta S_{n+k}\rangle \\ \cdots & & \cdots \\ \langle y, \Delta S_{n+k-1}\rangle & \cdots & \langle y, \Delta S_{n+2k-1}\rangle \end{vmatrix}}{\begin{vmatrix} 1 & \cdots & 1 \\ \langle y, \Delta S_n\rangle & \cdots & \langle y, \Delta S_{n+k}\rangle \\ \cdots & & \cdots \\ \langle y, \Delta S_{n+k-1}\rangle & \cdots & \langle y, \Delta S_{n+2k-1}\rangle \end{vmatrix}},$$

where $\langle\ ,\ \rangle$ is an inner product. Obviously, this formalism will work for any topological vector space $\mathscr{T}$: just pick y to be an element of the dual of $\mathscr{T}$.

It can be shown that the algorithm whose rules are

$$\varepsilon_{2k+1}^{(n)} = \varepsilon_{2k-1}^{(n+1)} + \frac{y}{\langle y, \varepsilon_{2k}^{(n+1)} - \varepsilon_{2k}^{(n)}\rangle}, \qquad k, n = 0, 1, \ldots,$$

$$\varepsilon_{2k+2}^{(n)} = \varepsilon_{2k}^{(n+1)} + \frac{\varepsilon_{2k}^{(n+1)} - \varepsilon_{2k}^{(n)}}{\langle \varepsilon_{2k+1}^{(n+1)} - \varepsilon_{2k+1}^{(n)}, \varepsilon_{2k}^{(n+1)} - \varepsilon_{2k}^{(n)}\rangle}, \qquad k, n = 0, 1, \ldots,$$

with initial conditions $\varepsilon_{-1}^{(n)} = 0$, $\varepsilon_0^{(n)} = S_n$, $n = 0, 1, 2, \ldots$, has the property that $\varepsilon_{2k}^{(n)} = e_k(S_n)$. I mentioned previously the lack of flexibility of the ε-algorithm. The same is true of the vector version, but there is a much more general algorithm, called the topological E-algorithm, that allows one to take advantage of information one may have about the shape of the sequence S_n.

In Chapter 6, the authors discuss the application of the material in the previous chapter to many problems in the applied sciences. It is here that this book is palpably stronger than any previous book. The applications are to summation of sequences and series, summation of double sequences, Chebyshev and Fourier series, continued fractions, vector sequences, systems of equations, projection methods, regularization and penalty techniques, nonlinear equations, continuation methods, eigenvalue and eigenvector computations, derivatives of eigensystems, integral and differential equations, implicit Runge-Kutta methods, boundary value problems, Laplace transform inversion, partial differential equations, interpolation and approximation, statistical procedures, in particular, Monte Carlo techniques, and numerical integration and differentiation. Many of the applications involve what are called mathematical *ill-posed problems*, that is, problems which are extremely sensitive to small perturbations in initial data.

Some numerical analysts consider sequence extrapolation to be a mere curiosity, hardly deserving of the energy its zealot admirers pour into it. No book on numerical analysis has an up-to-date account of the subject. The formulas (1), (2), (3) are among the most captivating in computational mathematics, and their theoretical and practical implications are considerable, yet how many numerical analysis books have an explanation of the ε-algorithm? I have long suspected that the agenda of those who write books on numerical analysis is not to enlarge our set of tools for solving difficult and important problems but merely to, well, write still more books on numerical analysis.

This is an absurd state of affairs. It reflects a know-nothingness that is puzzling in an age in which our personal health and welfare may depend on the resolution of ill-posed problems, such as inverse scattering problems (medical imaging) and problems in prediction theory (transportation of atmospheric pollutants). I do not believe that some new, magical method will appear to rescue us from the challenges these problems present. Our best hope for getting the information we need may be to use extrapolation methods, cleverly tailored to the context. Monte Carlo methods are the numerical court of last resort for many ill-posed problems, but the convergence of Monte Carlo methods, $O(n^{-1/2})$, n being the sample size, is so woeful that the error can swamp the computations, or the computations may be too responsive to the vagaries of the random-number generating scheme. Our knowledge about accelerating the convergence of statistical sequences is in its infancy. The methods we have now are not good, and are tainted with philosophical paradox. I think we can develop effective techniques, though they probably will depend on our ability to accurately characterize the distribution of the class of sequences being studied. We need to do much more research.

A unique feature of this book is a floppy disk containing subroutines for the practical implementation of extrapolation algorithms. In the last chapter, the authors describe the programs on the disk. They have tried to present the application side of the subject as more than just a mindless set of recipes. They emphasize that extrapolation methods must be programmed with great care, since often cancellation of significant digits is an inevitable concomitant to a method. Occasionally, one can use algebraic tricks to minimize numerical instability. Sometimes one cannot. By the way, I mentioned previously that nature is always kind to the numerical analyst: it never deceives by presenting the analyst with a spurious limit. The authors in this chapter give an example that forces me to qualify that statement. Consider the sequence S_n defined by

$$S_{n+1} = e^{-S_n}, \qquad n = 0, 1, 2, \dots, S_0 = 1.$$

The sequence converges, to $S = .567143\dots$. We know the Aitken Δ^2 method accelerates the convergence of this sequence, at least *theoretically*. Yet, if one retains only a *fixed* number of significant figures in the computation, say, 7, the Δ^2 method gives the spurious limit of $.5000000$!

The resources the authors provide the reader through this disk are really impressive: 25 methods are here, the most important methods in their scalar, vector, and topological manifestations.

Who should buy this book? Well, not just applied mathematicians. Anyone who uses numerical computations in the analysis of mathematical models of physical phenomena should own a copy. I emphasize that this is not a theoretical book, which places it in a different category from books like Delahaye's [2] or my own [6], and in its practical attributes—its expanse and user-friendliness—it far surpasses previous books by the first author. The theoretical content energizes the book, but the book is more of a hands-on manual in the craft of obtaining numerical data in onerous circumstances. There is nothing else available that does the job so well.

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25[65-06, 65D30, 65D32].—Terje O. Espelid & Alan Genz (Editors), *Numerical Integration: Recent Developments, Software and Applications*, NATO ASI Series, Series C: Mathematical and Physical Sciences, Vol. 357, Kluwer, Dordrecht, 1992, xii+367 pp., $24\frac{1}{2}$ cm. Price \$115.00/Dfl.195.00.

Although it is well over 2,000 years ago that Archimedes (287–212 B.C.) started the subject of numerical integration by finding approximations to π, the interest in the subject appears to be far from waning, and this NATO Advanced Research Workshop on Numerical Integration, held in Bergen, Norway over five days in June 1991, attracted 38 delegates from around the world. In all, 34 papers were presented, twenty five appear in full in this volume together with three extended abstracts and one note. The aim of the workshop was to survey recent progress and show how theoretical results have been used in software development and practical applications. This aim has been well achieved. The papers have been subdivided into four sections: "Numerical Integration Rules", "Numerical Integration Error Analysis", "Numerical Integration Applications", and finally "Numerical Integration Algorithms and Software". A complete list of authors and papers is given at the end of this review.

To the reviewer's delight, he found that this volume is dedicated to James Lyness "on the occasion of his 60th birthday". James has been contributing to the subject of numerical integration since his first paper, with John Blatt and David Mustard, was published nearly 30 years ago in the Computer Journal [3]. This Conference did not find James lacking and he describes, in good anecdotal style, his experience involving quadrature over a triangle or quadrilateral when the integrand has a known singularity at a vertex. Under an affine transformation of the region one can get disastrous results. James describes his experiences with this problem.

It is neither possible nor desirable for me to attempt to review, however briefly, every paper, so I shall make a highly personalized selection of papers for brief comment. Let me start with the papers by those authors who were also writing on numerical integration when James Lyness published his first paper. In 1967, Phil Davis and Phil Rabinowitz published the first edition of "Numerical Integration" [1]. This was followed in 1984 by a much enlarged second edition [2]. Both Phils were at this Conference, although Phil Davis' paper on "Gautschi Summation and the Spiral of Theodorus" does not appear in this volume. One can only speculate on the intricate mathematical threads the master was weaving on this occasion. Phil Rabinowitz, together with Will Smith, gave a careful analysis of interpolatory product integration in the presence of singularities both at end points of the interval of integration and at interior points.

It was in 1973 that Frank Stenger surprised us with his paper [4] in which he demonstrated that one can get exponential convergence of quadrature rules based on the trapezoidal rule even when the integrand has singularities. The analytic ideas given in that paper led to the so-called Sinc methods, which have been developed slowly over the years and deserve to be more widely used. Frank Stenger, together with Brian Keyes, Mike O'Reilly, and Ken Parker considered Sinc methods for indefinite integration and thereby extended their use to the solution $y' = F(x, y)$ over an arc $\widehat{ab}$ and $y(a)$ given. Bernard Bialecki, on the other hand, considered Sinc methods for the approximate evaluation of Cauchy principal value integrals. David Hunter described a method similar to the Sinc method for the case when the integrand has a singularity close to the interval of integration. In all these cases the errors behave like $a(N)\exp(-(bN)^{1/2})$, where a may depend (weakly) on N and $b > 0$. It is this exponential decay as $N \to \infty$ which makes these methods so attractive.

The section on error analysis had some excellent papers. Picking out just two of them, Walter Gautschi gave, as usual, a scholarly review of remainder estimates for analytic functions, and this was followed by Helmut Brass' beautifully written paper demonstrating the usefulness of approximation theory in this area.

These days, numerical integration for multi-dimensional integrals is of increasing importance, and the last decade has seen considerable growth in this area. It makes use of parts of mathematics which were not required for one-dimensional integrals. Ronald Cools gives an excellent survey of methods for constructing cubature formulae, making use of both invariant theory and ideal theory. This is followed by a paper by Karin Gatermann making use of linear representation theory of finite groups for constructing cubature formulae. Both Ian Sloan and Harald Niederreiter use number-theoretic methods in their respective papers as a basis for the construction of so-called lattice rules for the numerical integration of smooth periodic functions in s dimensions taken over the s-dimensional unit cube. Each paper contains a review of earlier work together with some recent developments undertaken by the respective authors.

But inevitably it is the software implementation of all these rules and associated error analyses which is of greatest interest to applied mathematicians, scientists, and engineers. The last 70 pages are devoted to this topic and of the papers in this section I might pick out for special mention just two. Ronald Cools again, this time with Ann Haegemans, gave a progress report on CUBPACK which, as its name implies, considers adaptive schemes for multi-dimensional integrals. Some good improvements in efficiency over existing schemes is reported for cases when the integrand is either singular or discontinuous. Finally, one of the editors, Terje Espelid, in a brief note reports on a new one-dimensional general-purpose algorithm DQAINT for adaptive quadrature of a function over a collection of intervals and compares his results with those obtained from other schemes.

Multi-dimensional integration and automatic integration occupied 20% and 10% respectively of [2]. This volume devotes about half of the papers to multi-dimensional integration and a further quarter to automatic integration. This points out the need for yet a third edition of the two Phils' magnificently organized and presented books. I understand that there are no plans for this at present, but perhaps a younger person might make his or her name in the field by undertaking this task in collaboration with the other two. There is excellent precedence for doing this with mathematical books. Two which come to mind are Watson in "Whittaker and Watson" and Jaeger in "Carslaw and Jaeger".

Browsing through these proceedings nearly 12 months after the event and from the other side of the world, one gets the impression that this has been an excellent Workshop, and the organizers are to be congratulated on their efforts. This volume does provide a good reference to the current state of the art and no self-respecting numerical analyst should be without access to these Proceedings. So if you cannot afford to put this volume on your own shelves, at least get your library to buy a copy.

Contributions: R. Cools, A survey of methods for constructing cubature formulae; K. Gatermann, Linear representations of finite groups and the ideal theoretical construction of G-invariant cubature formulas; H. J. Schmid and H. Berens, On the number of nodes of odd degree cubature formulae for integrals with Jacobi weights on a simplex; K.-J. Förster, On quadrature formulae near Gaussian quadrature; I. Sloan, Numerical integration in high dimensions—the lattice rule approach; H. Niederreiter, Existence theorems for effective lattice rules; B. Bialecki, SINC Quadratures for Cauchy principal value integrals; P. Rabinowitz and W. E. Smith, Interpolatory product integration in the presence of singularities: L_p theory; D. B. Hunter, The numerical evaluation of definite integrals affected by singularities near the interval of integration; N. I. Ioakimidis, Application of computer algebra software to the derivation of numerical integration rules for singular and hypersingular integrals; W. Gautschi, Remainder estimates for analytic functions; H. Brass, Error bounds based on approximation theory; K. Petras, One sided L_1-approximation and bounds for Peano kernels; R. Cariño, I. Robinson, and E. De Doncker, An algebraic study of the Levin transformation in numerical integration; G. Hämmerlin, Developments in solving integral equations numerically; C. Schwab and W. L. Wendland, Numerical integration of singular and hypersingular integrals in boundary element methods; J. N. Lyness, On handling singularities in finite elements; K. Hayami, A robust numerical integration method for 3-D boundary element analysis and its error analysis using complex function theory; J. Berntsen, On the numerical calculation of multidimensional integrals appearing in the theory of underwater acoustics; A. Genz, Statistics applications of subregion adaptive multiple numerical integration; F. Stenger, B. Keyes, M. O'Reilly, and K. Parker, The Sinc indefinite integration and initial value problems; P. Keast, Software for integration over triangles and general simplices; R. Cariño, I. Robinson, and E. De Doncker, An algorithm for automatic integration of certain singular functions over a triangle; R. Cools and A. Haegemans, CUBPACK: Progress report; E. De Doncker and J. Kapenga, Parallel cubature on loosely coupled systems; M. Beckers and A. Haegemans, Transformation of integrands for lattices rules; T. O. Espelid, DQAINT: An algorithm for adaptive quadrature over a collection of finite intervals; C. Schwab, A note on variable knot, variable order composite quadrature for integrands with power singularities; A. Sidi, Computation of oscillatory infinite integrals by extrapolation methods.

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1. P. J. Davis and P. Rabinowitz, *Numerical integration*, Blaisdell, Waltham, MA, 1967.
2. ——, *Numerical integration*, Academic Press, New York, 1984.
3. D. Mustard, J. N. Lyness and J. M. Blatt, *Numerical quadrature in n dimensions*, Comput. J. **6** (1963), 75–87.
4. F. Stenger, *Integration formulae based on the trapezoidal formula*, J. Inst. Math. Appl. **12** (1973), 103–114.

26[49-02, 49J15, 49M37].—K. C. P. MACHIELSEN, *Numerical Solution of Optimal Control Problems with State Constraints by Sequential Quadratic Programming in Function Space*, CWI Tract, Vol. 53, Centre for Mathematics and Computer Science, Amsterdam, 1988, vi+214 pp., 24 cm. Price: Softcover Dfl.59.00.

The aim of this book is to present the application of SQP (the sequential quadratic programming algorithm) to the optimal control of ordinary differential equations with state and control constraints.

The book is structured as follows. Chapter 1 is a brief introduction. Chapter 2 presents a theory of first- and second-order optimality conditions of abstract optimization problems in Banach spaces. The optimality conditions for optimal control problems are presented in Chapter 3, and Chapter 4 presents the principle of SQP for abstract optimization problems and optimal control (without discretization). Chapter 5 discusses the optimality conditions of the quadratic subproblems. The numerical resolution of the quadratic problems is discussed in Chapter 6; Chapter 7 presents some interesting examples in flight mechanics and servo systems. Final remarks are made in Chapter 8. The appendix is devoted to some numerical questions.

Dealing with the subject of this book is a difficult task because one has, on one hand, to present some results of optimization in infinite-dimensional spaces that use the most sophisticated tools of functional analysis, and on the other hand deal with some algorithmic and numerical questions which are rather involved. It is also worth noting that no other book (to the knowledge of the reviewer) covers such a vast domain.

The book seems to have been written very rapidly, and some drawbacks are apparent. For instance, the abstract optimization theory of Chapter 2 is very heavy, probably because of the desire of the author to present several qualification conditions. Also the second-order necessary condition of Theorem 2.14 says that the Hessian of the Lagrangian is nonnegative for a critical direction, a statement that is known to be false in general if an infinite number of inequality constraints appears. Some necessary second-order conditions taking this fact into account can be found in [1]. We note the following strange statement (after (3.2.1), p. 31): "$X = W_{1,\infty}[0, T]^n \times L_\infty[0, T]^n$ is assumed to be a Banach space, but it cannot be expected to be a Banach space unless the spaces $W_{1,\infty}[0, T^n]$ and $L_\infty[0, T]^n$ are both Banach spaces" (perhaps this must be understood as a joke).

On the other hand, Chapter 2 contains a useful review of the discussion of the smoothness of multipliers and of the "alternate optimality conditions" (involving the total derivative of state constraints). It is a merit of this book that it takes into account this "alternative theory" that explicitly handles the switching points (i.e., the times at which the set of active constraints change) and is usually associated with shooting methods. Active set strategies for the resolution of the quadratic subproblems are also discussed.

The examples are interesting. They present the numerical solution of several nontrivial real-world optimal control problems with first- and second-order state constraints.

In conclusion, this book should be considered as an introduction to the subject rather than a definitive treatise. Nevertheless, it should interest anybody willing to have an overview of some modern approaches to numerical optimal control.

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1. H. Kawasaki, *The upper and lower second order derivative for a sup-type function*, Math. Programming **41** (1988), 327–339.

27[90C20, 90C30, 65K10].—Christodoulos A. Floudas & Panos M. Pardalos (Editors), *Recent Advances in Global Optimization*, Princeton Series in Computer Science, Princeton Univ. Press, Princeton, NJ, 1992, x+663 pp., $23\frac{1}{2}$ cm. Price \$69.50 hardcover, \$39.50 paperback.

This volume contains refereed versions of twenty-seven papers presented at the "Recent Advances in Global Optimization" conference held at Princeton University in May, 1991. If you have an interest in global optimization, then you will find this book fascinating. Its pages reveal the breadth of current research in global optimization, from methods tailored to the optimal reloading of a nuclear power plant, to methods designed to minimize portfolio risk.

Global optimization involves finding the minimum (or maximum) value assumed by a continuous real-valued objective function of many variables, over a constraint set which is generally assumed to be compact. Authors differ over the structure assumed of the objective function, and of the constraints. Broadly speaking, the early papers in this volume assume a lot of structure, while the later papers assume little.

The first seven papers consider variants of the situation in which the objective function is quadratic. Complexity questions are addressed, as well as applications of quadratic programming to the finding of Hamiltonian cycles in directed graphs and minimum-cost solutions to network flows. Two papers are devoted to the Linear Complementarity Problem. Interior-point methods are developed by Kamath and Karmarkar for finding upper bounds to the convex quadratic problem, when the domain is the set of vertices of a hypercube.

Papers in the next bracket of four assume a special form for the objective function, generally related to convexity. An example is the paper of Tuy and Al-Khayyal. This considers the problem of maximizing a sum of functions, each of which is a composition of a decreasing convex function with a convex function. They show that their situation is equivalent to a convex maximization problem, a classical problem in the subject.

Functions which are built from ratios of linear functions are dealt with in the next two papers. Falk and Palocsay, motivated by a shipping scheduling context, develop a new method for optimizing the sum of two such functions. Konno and Yajima optimize the product of two ratios of linear function, motivated by a bond portfolio problem.

Lipschitz-based methods are covered in three papers. Evtushenko, Potapov, and Korotkich survey nonuniform space-covering techniques. Mladineo initiates a study of an algorithm which combines her cone algorithm with Pure Random Search. Pinter describes several existing and potential areas of application of Lipschitz methods.

Standing apart is the paper of Moore, Hansen, and Leclerc. This is a pleasure to read, and provides an excellent overview of the interval-arithmetic approach to global optimization.

Three papers involve a random flavor. Zabinsky and co-authors present an algorithm, based on the hit-and-run technique, which aims to realize Pure Adaptive Search. Törn and Viitanen introduce a new clustering algorithm. Shalloway presents a deterministic annealing algorithm which aims to converge to the global minimum with few function evaluations.

A paper of appeal to the functional analyst is that due to Zheng and Zhuang. They approximate an infinite-dimensional constrained optimization problem with a sequence of finite-dimensional problems.

Discrete-valued constraint variables are discussed. Grossmann, Voudouris, and Ghattas address engineering problems involving components which come in standard sizes only. Nielsen and Zenios, motivated by the problem of designing a computer board so as to minimize the total length of connecting wiring, develop a method to solve a mixed-integer nonlinear work program with generalized network constraints.

Least structure of all is assumed in the paper of Li, Pardalos, and Levine. They consider the problem of optimal nuclear reactor design, where the aim is to reach an optimal state at the end of a fuel cycle, subject to constraints, such as maintaining a certain level of energy production.

Trust region methods of Powell come under scrutiny, while a new path-following method is presented to find all the stationary points of a nonlinear program with inequality constraints.

The final paper is intriguing. Barbagallo, Recchioni and Zirilli tackle the question "Does your software do what it is supposed to do?" They set up the language of a Warnier's flow chart and investigate the problem of finding the n test cases which will maximize the reliability of the software.

In summary, this set of papers is compulsory reading for the researcher in global optimization, and highly recommended for those interested in gathering a flavor of the methods and applications of the subject.

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28[49-01, 49-04, 65K10, 90C35].—DIMITRI P. BERTSEKAS, *Linear Network Optimization: Algorithms and Codes*, The MIT Press, Cambridge, MA, 1991, xi+359 pp., $23\frac{1}{2}$ cm. Price $39.95.

This book provides an introduction to the field of network optimization. The most general problem treated in the book is the *minimum-cost flow problem*, which is the problem of finding a minimum-cost flow in a network that satisfies supplies and demands at the nodes, and upper and lower bounds on the arcs. The book also treats the standard special cases of this problem, namely the assignment problem, the maximum-flow problem and the shortest-path problem. The book consists of five chapters and an appendix. Chapter 1 serves as an introduction, both to problem types and algorithmic strategies. Each of Chapters 2–4 describes an algorithmic strategy in detail; Chapter 2 focuses on the simplex methods, Chapter 3 on dual ascent methods, and Chapter 4 on auction methods. Chapter 5 provides a brief overview of the empirical performance of these methods. Finally, the appendix consists of FORTRAN listings of many of the algorithms discussed in the text.

The book is interesting for what it does, as well as for what it does not do. The book is not comprehensive, nor does it pretend to be. As the author indicates in the preface, the coverage is focused and selective. The primary algorithmic treatment of the maximum-flow and shortest-path problems is done in the introduction. The treatment of the maximum-flow problem is cursory, at best. Little mention is made of the recent preflow-push algorithms. The shortest-path problem receives a more detailed treatment, concentrating on the single-source multiple-destination version of the problem. The treatment is standard. Chapter 2, entitled *Simplex Methods*, is devoted almost entirely to a description of the primal simplex method for the minimum-cost flow problem. Again, the description is standard. Chapter 3, entitled *Dual Ascent Methods*, first describes the primal-dual method for the minimum-cost flow problem and then the relaxation method of the author. The primal-dual method is given both in its basic form and as a sequential shortest-path method. What sets this book apart from others is Chapter 4, entitled *Auction Algorithms*. This chapter is roughly twice as long as the previous two and provides an in-depth presentation of auction algorithms, which were first proposed by the author. The chapter begins by describing an auction algorithm for the assignment problem, which seems to be its most natural domain. The chapter then winds its way through variations of the basic auction algorithm applied to variations of the assignment problem. In the end, an auction algorithm for the minimum-cost flow problem is given.

The book is well written. The algorithms are motivated both through examples and intuitive reasoning. Proofs of correctness are included throughout.

The author points out useful relationships between various algorithms. Data structures needed to implement each algorithm are briefly discussed. For the most part, the book avoids worst-case complexity issues, opting instead for brief discussions of the empirical performance of the algorithms. The book is appropriate for an introductory graduate-level course. It contains a good collection of exercises. It is more or less self-contained; however, some knowledge in linear programming would be useful.

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29[68–00, 68Q40].—MARTHA L. ABELL & JAMES P. BRASELTON, *The Mathematica Handbook*, Academic Press, Boston, 1992, xvi+789 pp., $23\frac{1}{2}$ cm. Price: Softcover $32.50.

Intended as a supplement to the manual for the Macintosh version of the computer system "Mathematica" distributed by Wolfram Research Inc., this handbook is organized alphabetically rather than by topic. Its primary strength is that it provides many simple examples covering some 1500 commands. Virtually every page has one or more Macintosh computer bit-map displays.

Unfortunately, the book is typographically cluttered, the bit-map displays detract from the readability, it has not been carefully proofread, and it does not explain any of the numerous topics that are likely to remain unclear from the manual. I noted particularly inadequate coverage of such confusing subjects as Block, Module, Context, If, Function, and Patterns.

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30[68Q40].—MALCOLM A. H. MACCALLUM & FRANCIS J. WRIGHT, *Algebraic Computing with REDUCE*, Lecture Notes from the First Brazilian School on Computer Algebra, Vol. 1, Clarendon Press, Oxford, 1991, xx+294 pp., $23\frac{1}{2}$ cm. Price $59.95 hardcover, $29.95 paperback.

The REDUCE Computer Algebra system has a long history of wide distribution on a variety of computers. Its international community continues to use and improve the program, under the coordination of its original author, A. C. Hearn at the RAND Corp.

This text, which is based on a series of lectures on REDUCE delivered in 1989, targets an audience of persons who need more information than is readily available from the REDUCE manual and the source code for the system.

The authors provide authoritative and substantial additional background, commentary, and examples of usage and programming using REDUCE data types and commands. The authors' concerns range from the mundane (e.g., the differences between the ATARI ST version and other systems) to deep mathematical issues (at least briefly, the algorithms for polynomial factoring and indefinite integration are discussed).

Strengths of the book include the demystification of some components of REDUCE as well as its implementation in LISP, and pointers to research papers and books with further details.

This text is recommended for serious REDUCE users as well as for the casual REDUCE user interested in learning more about the system.

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31[65–06, 68–06].—ROBERT E. O'MALLEY, JR. (Editor), *ICIAM* 91: *Proceedings of the Second International Conference on Industrial and Applied Mathematics*, SIAM, Philadelphia, PA, 1992, xviii+391 pp., 26 cm. Price $61.50.

The conference in the title, sponsored internationally by 12 societies of Applied, Industrial and Computational Mathematics, was held July 8–12, 1991, in Washington, D.C. Part I of the proceedings contains the text of 17 invited presentations, Part II an account of over 160 minisymposia organized in 29 chapters according to subject areas.

The authors and titles of the invited papers in Part I are: J. M. Ball, Dynamic energy minimization and phase transformations in solids; G. I. Barenblatt, Intermediate asymptotics in micromechanics; M. Brady, Computer vision: mathematics and computing; R. Coifman, Y. Meyer & V. Wickerhauser, Adapted wave form analysis, wavelet-packets and applications; A. R. Conn, N. Gould & Ph. L. Toint, Large-scale nonlinear constrained optimization; C. N. Dawson & M. F. Wheeler, Time-splitting methods for advection-diffusion-reaction equations arising in contaminant transport; W. Eckhaus, On modulation equations of the Ginzburg-Landau type; A Fasano, Modelling the solidification of polymers: an example of an ECMI cooperation; M. Grötschel, Discrete mathematics in manufacturing; F. L. Chalot & T. J. R. Hughes, Analysis of hypersonic flows in thermochemical equilibrium by application of the Galerkin/least-squares formulation; N. Karmarkar, Interior-point methods in optimization; P. L. Lions, Viscosity solutions and optimal control; M. Mimura, Dynamics of patterns, waves, and interfaces from the reaction-diffusion aspect; J. D. Murray, Complex pattern formation in embryology: models, mathematics, and biological implications; G. Ruget, Trends in radar architectures; D. J. Wallace, Massively parallel computing: status and prospects; H. Yserentant, Hierarchical bases.

A useful feature in Part II is a list of suggested reading appended to each chapter.

The volume, which is attractively sprinkled with photographs of speakers and participants, ends with an author index and a list of attendees.

W. G.

32[65C10, 68Q15, 94A60].—NOAM NISAN, *Using Hard Problems to Create Pseudorandom Generators*, An ACM Distinguished Dissertation 1990, The MIT Press, Cambridge, MA, 1992, x+43 pp., $23\frac{1}{2}$ cm. Price $20.00.

This book is a slightly revised version of the author's doctoral dissertation written under the supervision of R. Karp at Berkeley. It deals with pseudorandom bit generation for cryptographic purposes in the sense of A. Yao, M. Blum, and S. Micali, that is, the problem of finding functions which stretch a short string of truly random bits into a long string of bits which looks random to observers having limited computational power. This problem is closely linked with issues of computational complexity. The author presents two different constructions of pseudorandom bit generators that relate to given complexity classes. The first construction is of a general type and produces bit strings that look random to any algorithm from a complexity class C using an arbitrary function that is hard for C. In particular, using the known lower bounds for constant-depth circuits, this construction yields unconditionally proven pseudorandom bit generators for constant-depth circuits. The second construction, which does not rely on any unproven hypothesis, produces bit strings that look random to all Logspace machines.

H. N.

33[65C05, 65D32].—Nancy Flournoy & Robert K. Tsutakawa (Editors), *Statistical Multiple Integration*, Contemporary Mathematics, Vol. 115, American Mathematical Society, Providence, RI, 1991, xii+276 pp., $25\frac{1}{2}$ cm. Price \$71.00.

This collection of articles arose from the AMS–IMS–SIAM Joint Summer Research Conference on Statistical Multiple Integration which was held at Humboldt University, Arcata, California, in 1989. The emphasis in these papers is on Monte Carlo methods and analytic approximation methods. Articles of particular interest for numerical analysts are those by D. K. Kahaner on existing software for multidimensional numerical integration, by A. Genz on subregion adaptive algorithms, by M. Mascagni on the implementation of algorithms for high-dimensional numerical integration on massively parallel computers, and by M.-S. Oh on importance sampling.

H. N.

34[11–06, 11B37, 11B39].—G. E. Bergum, A. N. Philippou & A. F. Horadam (Editors), *Applications of Fibonacci Numbers*, Vol. 4, Kluwer, Dordrecht, 1991, xxiv+313 pp., $24\frac{1}{2}$ cm. Price \$99.00/Dfl.180.

This book consists of thirty-three papers from among the thirty-eight papers presented at the Fourth International Conference on Fibonacci Numbers and their Applications held at Wake Forest University, Winston-Salem, NC from July 30–August 3, 1990. The theme of these Conferences is wider than that suggested by their title; in fact, they are devoted to the theory and application of linear recurring sequences in general. The papers are very diverse, discussing the occurrence of these sequences in such settings as: algebra, combinatorics, graph theory, geometry, number theory, probability, and even electronics.

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