

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

13[65-06, 65N55].—W. HACKBUSCH & U. TROTTEBERG (Editors), *Multigrid Methods III*, Internat. Ser. Numer. Math., Vol. 98, Birkhäuser, Basel, 1991, xi+394 pp., 24 cm. Price \$98.00.

This volume contains seven papers based on invited addresses and twenty-three papers selected from seventy-six papers presented at the conference. A complete list of titles of the rest of the papers presented at the conference is given at the end of the volume.

J. H. B.

14[65-06, 65N22, 65N30].—DAVID E. KEYES, TONY F. CHAN, GÉRARD MEURANT, JEFFREY S. SCROGGS & ROBERT G. VOIGT (Editors), *Domain Decomposition Methods for Partial Differential Equations*, SIAM Proceedings Series, SIAM, Philadelphia, PA, 1992, xiv+623 pp., 25 cm. Price: Softcover \$74.00.

This volume contains papers presented at the Fifth Conference on Domain Decomposition Methods for Partial Differential Equations held in Norfolk, Virginia, in May of 1991. It consists of four parts. Part I, entitled "Theory", contains twelve papers, including analyses of so-called Schwarz methods, methods for nonselfadjoint problems and the biharmonic Dirichlet problem. The second part concerns algorithms and contains seventeen papers. Part III is about parallel implementation issues, with six papers on this subject. Finally, the last part contains nineteen papers on applications of domain decomposition methods.

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15[11D25, 11Y50].—KENJI KOYAMA, *Tables of solutions of the Diophantine equation $x^3 + y^3 + z^3 = n$* , 57 pages of tables and 3 pages of introductory text, deposited in the UMT file.

A computer search has been made for solutions of the equation $x^3 + y^3 + z^3 = n$ in the range $\max(|x|, |y|, |z|) \leq 2097151$ and $0 < n \leq 1000$. We have discovered 18 new integer solutions for $n \in \{39, 143, 180, 231, 312, 321, 367, 439, 516, 542, 556, 660, 663, 754, 777, 870\}$. As a result, there are 54 values of n (except $n \equiv \pm 4 \pmod{9}$) for which no solutions are found. Table 2

(2 pages) contains the number of primitive solutions found for each of 768 values of n . Table 1 (55 pages) contains the actual values of primitive solutions (x, y, z) for each n .

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16[42C10, 65N30, 65N35].—DANIELE FUNARO, *Polynomial Approximation of Differential Equations*, Lecture Notes in Phys., Vol. 8, Springer, 1992, x+305 pp., 24½ cm. Price \$46.00.

This book deals with approximate solutions of Differential Equations (DEs) which are based on spectral methods of polynomial type.

The framework is as follows. Let $u(x)$ denote the exact solution of the DE, $L(x, u, Du, \dots) = F$. The purpose is to compute an approximation to a *polynomial projection* of this solution, $P_n u(x)$. To this end, one computes an approximate solution, $u_N(x)$, by solving the nearby finite-dimensional model $L_N(x, u_N, D_N u_N, \dots) = F_N$. This framework is a synthesis of several ingredients: the polynomial projections, P_N , and the quality of their approximation of the identity, $\|I - P_N\|$, the derivative matrices, $D_N = P_N D P_N$, and the integration of these ingredients into DEs together with their initial and boundary conditions.

The first four chapters are concerned with the underlying *polynomial projections*, P_N . A distinctive character of spectral methods is the use of *global* projections. In the present context such projections arise as truncated Fourier-Jacobi expansions. After the introductory material in Chapter 1 on Legendre, Chebyshev, and other prototype Jacobi polynomials, Chapter 2 proceeds with a discussion of their weighted orthogonality and the basic properties related to their Fourier expansions. Chapter 3 treats the corresponding projections based on Gauss quadratures. Here, moments of the Galerkin-type Fourier projections of Chapter 2 are replaced by the closely related interpolatory projections. These discrete projections enable pointwise multiplication and composition in physical space, and Chapter 4 complements this with a description of fast transforms from and to the dual Fourier space.

The next two chapters focus attention on *approximation errors*. The key question here is how well the exact solution, u , is represented by its polynomial projection, $P_n u$. The results in Chapter 6 amplify yet another distinctive feature of spectral projections: measured in terms of appropriate function spaces introduced in Chapter 5, the error $\|(I - P_N)u\|$ decays as fast as the global smoothness of u permits (popularly known as *spectral accuracy*). Chapters 7 and 8 study the “discrete” derivative matrices, $P_N D P_N$, associated with various projections and different assignments of boundary conditions.

The construction of polynomial approximations for ordinary DEs is the topic of Chapter 9; time-dependent problems governed by partial DEs are discussed in Chapter 10. Special attention is paid to *weak formulations* which yield effective treatments of boundary conditions, and to stability estimates—properly weighted—which in turn yield *convergence rate* estimates for $\|u_N - P_N u\|$.

Chapter 11 is devoted to domain decomposition methods which, in the present context, are used to decouple the global nature of spectral methods into independent subdomain computations. Finally, the book concludes with several examples in Chapters 12–13.

The book concentrates mainly on the analysis of polynomial methods. This enables the author to expand the presentation of the corresponding material in the comprehensive treatment of spectral methods of [1, §§9–10]. The self-contained exposition of the material and an easygoing style of writing make this an excellent textbook to accompany a course on spectral methods. At the same time, it is a very well-written scholarly book that will serve both practitioners and experts. There is no doubt that *Polynomial Approximation of Differential Equations* is an important addition to the contemporary literature on spectral methods.

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1. C. Canuto, M. Y. Hussaini, A. Quarteroni, and T. Zang, *Spectral methods in fluid dynamics*, Springer Series in Computational Physics, New York, 1988.

17[65–06, 65Kxx, 65H10].—EUGENE L. ALLGOWER & KURT GEORG (Editors), *Computational Solutions of Nonlinear Systems of Equations*, Lectures in Appl. Math., Vol. 26, Amer. Math. Soc., Providence, RI, 1990, xx+762 pp., 23½ cm. Price \$235.00.

This volume contains articles presented at the 1988 AMS-SIAM Summer Seminar on “Computational Solution of Nonlinear Systems of Equations.” There are a total of 40 papers covering a wide range of topics on the numerical solution of nonlinear equations. A collection of nonlinear test problems, compiled and edited by Jorge Moré, is also included.

In the area of continuation methods, the papers cover homotopy methods, PL algorithms, smooth penalty functions, and nonsingular polynomial continuation. Results of these continuation algorithms on variational problems, constrained optimization, two-point boundary value problems, and design problems are also included.

Newton and quasi-Newton methods are discussed for undetermined systems, for nonlinear convection diffusion equations, and for the linear complementarity problems. Duality algorithms and low-storage methods for unconstrained optimization are included.

Specific applications such as contaminant transport in porous media, dielectric spectroscopy, and collisional kinetic equations are also treated.

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