REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

8[65–02, 65F10, 65L05, 76D05, 65N99].—Guri I. Marchuk (Editor), *Numerical Methods and Applications*, CRC Press, Boca Raton, FL, 1994, x + 272 pp., 24 cm. Price $59.95.

This volume contains six articles by leading Russian numerical analysts working in Moscow. They are selected and translated, in some cases in slightly modified form, from Vol. 8 in a series entitled *Numerical Processes and Systems* and which is associated with the Institute of Numerical Mathematics of the Russian Academy of Science. Even though the quality of presentation and translation is somewhat uneven, the volume gives a good overview of the activity that is presently pursued in numerical mathematics in Moscow. We refrain from reviewing the papers individually but indicate the scope of the volume by reproducing its table of contents:

V. T.

9[65L05].—B. P. Sommeijer, *Parallelism in the Numerical Integration of Initial Value Problems*, CWI Tracts, Vol. 99, Centrum voor Wiskunde en Informatica, Amsterdam, 1993, vi + 195 pp., 24 cm. Price: Softcover Dfl.50.00.

During 1988, the Centrum voor Wiskunde en Informatica, in Amsterdam,

0025-5718/95 $1.00 + $.25 per page

initiated a research program on parallel methods for ordinary differential equations. This tract is a compilation of six papers [1, 2, 3, 4, 5, 6] coming out of this research, together with an extended introduction by B. P. Sommeijer. The introduction is more than a preamble to the rest of the monograph, and is, in essence, a further paper. The aspect of parallelism considered here, "across the method", makes sense only for multistage methods. Explicit Runge-Kutta methods, in their traditional formulation, have little to offer, but there seem to be reasonable prospects for iterated Runge-Kutta methods and for the type of multivalue multistage methods described here as block Runge-Kutta methods. For stiff problems, implicit Runge-Kutta methods can sometimes be effectively implemented using parallel iterations. To all these options, methods generalizing predictor-corrector methods can be added. The papers included here discuss many aspects of these many method types, such as order, convergence of iterations, stability, local error estimation and stepsize control. The theoretical work is supported by insightful numerical evaluations and comparisons. For both nonstiff and stiff problems, and for the related differential-algebraic equations, parallelism across the method is still a vital area of research and the work presented in this tract is a valuable introduction to this topic.

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1. P. J. van der Houwen and B. P. Sommeijer, *Parallel iteration of high-order Runge-Kutta methods with stepsize control*, J. Comput. Appl. Math. **29** (1990), 111–127.

2. ——, *Block Runge-Kutta methods on parallel computers*, Z. Angew. Math. Mech. **72** (1) (1992), 3–18.

3. B. P. Sommeijer, W. Couzy, and P. J. van der Houwen, *A-stable parallel block methods for ordinary and integro-differential equations*, Appl. Numer. Math. **9** (1992), 267–281.

4. P. J. van der Houwen, B. P. Sommeijer, and W. Couzy, *Embedded diagonally implicit Runge-Kutta algorithms on parallel computers*, Math. Comp. **58** (1992), 135–159.

5. P. J. van der Houwen and B. P. Sommeijer, *Iterated Runge-Kutta methods on parallel computers*, SIAM J. Sci. Statist. Comput. **12** (1991), 1000–1028.

6. ——, *Analysis of parallel diagonally implicit iteration of Runge-Kutta methods*, Appl. Numer. Math. **11** (1993), 169–188.

10[65L15, 65Y15, 34B24].—John D. Pryce, *Numerical Solution of Sturm-Liouville Problems*, Monographs on Numerical Analysis, Oxford Univ. Press, Oxford, 1993, xiv + 322 pp., 24 cm. Price $56.50.

The past few years have seen a remarkable production of mathematical software for Sturm-Liouville problems; all of these codes treat various fairly wide classes of Sturm-Liouville problems (both regular and singular) with automatic error control of some kind. The timely text of Pryce discusses all of these codes (as well as other numerical methods) and, more importantly, provides considerable detail on the underlying mathematics. In the words of the author "It

[Sturm-Liouville software] is an area, like the numerical analysis of partial differential equations, of which it can be considered a small part, where a pervasive role is played by deep classical and functional analysis." The sophistication of these modern software packages is such that a thorough understanding of the related mathematics is crucial if one wants to fully understand them.

The initial chapter of the book contains introductory material on the mathematical notation and terminology to be used along with some examples of how Sturm-Liouville problems can arise in applications. The second chapter provides background on the mathematical theory needed to study these problems, including standard transformations and asymptotic developments that are used to study the qualitative behavior of eigenvalues and eigenfunctions. Besides giving insight into the theory, much of this material is (or should be) incorporated into the various software packages. The third and fourth chapters introduce the classical numerical techniques of finite differences and variational methods. While these have not been successfully incorporated into mathematical software (for a variety of reasons discussed in the text, including inefficiencies at large eigenvalue indices and in estimating global errors), they do shed light on some of the numerical problems associated with Sturm-Liouville calculations. Also, some research effort has been given to overcoming some of their shortcomings. Chapters 5 and 6 address the two methods underlying the recently released software packages: shooting (after a Prüfer transformation) and coefficient approximation (which the author graciously calls Pruess methods, though, as the author makes clear, only the theory, not the algorithm, originated with this reviewer). Great care is taken with the mathematics underlying the codes. Chapter 7 presents the relevant theory for singular problems, the most likely to be encountered in applications. How the numerical methods are adapted to these singular cases is the focus of Chapter 8. Chapter 9 discusses the computation of Sturm-Liouville eigenfunctions whose efficient and accurate calculation is still not as well developed as that for eigenvalues. Chapter 10 is devoted to the computation of physical resonances, another area where the author has made significant contributions. There is an assortment of topics in Chapter 11, including the use of interval arithmetic, multipoint and vector problems, and the calculation of the spectral density function. The final chapter contains a dozen challenges for future research, both theoretical and computational. A discussion of benchmarking and a long list of problems are contained in a second appendix.

To this reviewer, the major strength of the book is the thoroughness with which the author presents both theoretical and numerical material related to Sturm-Liouville calculations. Of course, one is always at some risk describing "current" software, as there are always updates and upgrades occurring, which quickly render any printed description out of date. For instance, there are several places in the text regarding SLEDGE where the author's comments, while valid for the version he had, no longer hold. This may be true for his own NAG codes as well! Another strength is the numerous problem sets throughout the text, some of which are challenging; a graduate course on this specialized material could be self-contained if this text were used. Finally, the text is a must for any current or prospective researcher who is interested in Sturm-Liouville calculations, and it is strongly recommended for anyone interested in developing serious mathematical software to see the interplay between theory and implementation.

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11[65–02, 68–02, 68U07].—JOSEF HOSCHEK & DIETER LASSER, *Fundamentals of Computer Aided Geometric Design* (translated from the German by Larry L. Schumaker), A K Peters, Wellesley, MA, 1993, xviii + 727 pp., $23\frac{1}{2}$ cm. Price \$79.95.

We have here a fine addition to the textbook literature in computer-aided geometric design (CAGD). The book evolved in several stages from lecture notes (in German) prepared in 1986. The first published edition appeared in German under the Teubner imprint, and the second German edition appeared in 1992. Larry Schumaker undertook the translation and typesetting that resulted in the edition reviewed here.

The text gives a systematic account of CAGD, and is very much up-to-date. In a short review, we can only outline the contents chapter-by-chapter. Chapter 1 discusses transformations, projections, visibility methods, shading and reflection. Chapter 2 reviews elementary differential geometry, classical interpolation methods, and least squares procedures. Chapter 3 is about splines, including tension splines and exponential splines, all in one variable. Chapter 4 gives the theory of B-splines and Bézier curves. This occupies almost 100 pages. Chapter 5 continues with more technical spline topics: "FC" continuity, curvature continuity, Manning's splines, tau-splines, etc. Chapter 6 begins the study of spline surfaces, including tensor-product surfaces and splines on triangulations. Chapter 7 continues the study of surfaces, including multi-patch methods. Chapter 8 is devoted to Gordon-Coons surfaces (blending methods). Chapter 9 deals with scattered data interpolation by several procedures, including radial basis methods, Shepard's method, and methods from the finite element realm. Chapter 10 is devoted to basis transformations for the representation of curves and surfaces. In Chapter 11, methods for problems in high dimensions are discussed. The last five chapters deal with intersections of curves and surfaces, smoothing techniques, blending methods, offset curves and surfaces, and applications to milling processes.

The book has 727 pages, a bibliography of 83 pages, and a wealth of figures (approximately one per page). The authors (and translator) are to be congratulated for producing a comprehensive book on a timely topic.

E. W. C.

12[68–01, 68U05].—JOSEPH O'ROURKE, *Computational Geometry in C*, Cambridge Univ. Press, Cambridge, MA, 1994, xii + 346 pp., 26 cm. Price \$59.95 hardcover, \$24.95 paperback.

Computational geometry could be an ideal subject for undergraduate study. Students of mathematics would see how challenging it is to apply mathematical ideas in a world where there is no real arithmetic, where common assumptions of "general position" or "nondegeneracy" can severely limit or completely destroy the usefulness of a program, and where "abuse of notation" only invites abusive error messages in return. Students of computer science would see applications of the calculus and linear algebra classes that (to many) represent only suffering, the importance of proof to making an algorithm work, and some advantages and disadvantages of asymptotic analysis of algorithm performance.

The textbooks available until now, however, including [1, 2, 3], have been aimed at graduate students. Taking a "high-level" approach to the subject, at best they present "pseudocode" solutions to problems, and they usually ignore the treatment of geometric degeneracies. The preface to the present text says that it is suitable for undergraduates, especially junior and senior majors in computer science or mathematics, and the "in C" in its title implies that it contains useful programs that implement algorithms from computational geometry.

Chapters 1–6 cover problems central to computational geometry, including computing polygon partitions, convex hulls, Voronoi diagrams, and arrangements. I would classify most of the problems in Chapters 7 and 8, on geometric search and intersection, and motion planning, as "advanced topics" in computational geometry. Section 8.6, on robot arm motion, is exceptional: it conveys to the reader both an appropriate level of mathematical and computational detail and the author's enthusiasm for the topic. Would that the more fundamental material of the first six chapters had been presented with such care.

Publishing code requires considerable bravery on the part of an author. Published code reveals to everyone all the details its author either considered or failed to consider. Even when published code is correct, readers are bound to criticize the way it is organized, the choice of data structures and algorithms, the names of variables, and the indentation convention. While some of my criticisms do indeed fall into these categories, most of them reflect larger questions about overall programming judgment. (It is disappointing to note, however, that the programs were typeset by hand, and many include typographical errors introduced by the printers. It is possible to include program source text directly from the version that was compiled and tested, which would eliminate one source of error.)

Chapter 1 begins with some useful small functions to compute areas, point orientations, and the intersections of simple curves (line segments and circles). Mathematicians may consider these functions trivial, but computer programmers know how hard it is to make them work when areas may vanish, points may coincide, or other degeneracies may occur.

Most of the larger programs, however, are disappointing. The preface warns that the text includes implementations of only nine algorithms, but even those nine programs are mostly toys and curios.

(1) The first program triangulates a polygon by chopping off one triangle at a time, so it runs in quadratic time. This choice of a simple algorithm is reasonable since it illustrates nicely the meticulous care required to implement reliable geometric computations. Unfortunately, the program stores a polygon in an array with room for up to 1000 vertices, wasting both space and time: Since the algorithm works by repeatedly deleting vertices from the middle of the array, this choice of data structure is simply wrong. O'Rourke repeatedly defends the use of arrays in the name of making the code easier to understand.

In my experience, upper-division undergraduates can cope with linked data structures.

(2) The program to compute a Delaunay triangulation runs in quartic time, which the text admits is "rarely acceptable." It would have been far better to present an implementation of the incremental algorithm (the most popular, according to the text), which runs in quadratic time.

(3) The program to test whether a given point lies inside a given polygon could be one of the cleanest of the long programs in the book. As written, however, it destroys the vertices of the input polygon, though a footnote admits that this effect is "rarely desired."

(4) Of the program to compute the intersection of two convex polygons, "It only remains to settle the 'degenerate' cases to get code. We will not argue these details here but only claim that the algorithm is fortunately robust in that almost any decision works" (p. 246). Just *where* could it *possibly* be *more* appropriate to argue the correctness of a C program implementing a geometric algorithm than in a book entitled *Computational Geometry in* C?

Of course not every algorithm in the text needs to appear in an implementation. Nevertheless, I consider it a serious mistake not to have included any implementation of a plane-sweep algorithm. The text itself includes ample justification for the importance of the plane-sweep paradigm in connection with triangulations, Voronoi diagrams, and arrangements.

Since the book contains only nine implementations, much of the remainder is written in the style of a mathematics book, with definitions, lemmas, theorems, and proofs. Mathematicians who criticize computer scientists' attempts at writing mathematics as clumsy and exasperating will not be heartened. Almost every proof asserts that something is "clear" or "evident." This lapse is especially surprising since there are so many examples of discredited results in computational geometry that were once said to be "clear" or "proven by picture."

O'Rourke is fond of exploring questions by drawing pictures, and he can do a nice job of bringing us along on these experimental forays, both building our intuition and showing us pitfalls. Unfortunately, he often fails to establish a point beyond which the experiments are over, where he states the definitions he will use or the results he will prove.

(1) "A little experimentation can lead to a conviction that no more than two [guards] are even needed, so that $G(6) = 2$" (p. 5). Are we meant to believe that $G(6) = 2$ because of our experiments, our conviction, or the proof several pages later?
(2) The first paragraph of §3.2.2 explains what it means for an edge of a polygon to be extreme. Then the second paragraph begins "There is unfortunately an inaccuracy in our characterization of extreme edges" (p. 74), and goes on to explain a modification. Just what, finally, is "our characterization"?
(3) The proof of Euler's formula on p. 119 does not distinguish appropriately between a planar graph and a particular embedding of it.
(4) In a proof, "Because no points are on the boundary of $C(x)$ other than a and b (by hypothesis), there must be freedom to wiggle x a bit and maintain emptiness. In particular, we can move x along B_{ab}, the bisector between a and b, and maintain emptiness while keeping

the circle through a and b" (p. 177). The mention of x "wiggling" should be supplemented by a formal statement that "there exists $\varepsilon > 0$ such that $C(z)$ is empty whenever z is chosen such that $z \in B_{ab}$ and $|z - x| < \varepsilon$." When will students ever learn to express concepts like "wiggle" appropriately if textbooks don't show them?

Let me emphasize that I *do* appreciate it when books include explorations and intuitive explanations. But my experience with upper-division undergraduates suggests that they also need explicit and careful statements of results and their proofs.

A couple of minor errors in the mathematical exposition are especially apt to confuse students.

(1) The attempt to define a polygon without appeal to topology (p. 1) is incorrect: the condition that nonadjacent edges of the polygon do not intersect should be written "$e_i \cap e_j = \varnothing$ whenever $i - 1 \neq j \neq i + 1$."

(2) On p. 74, the notation "$l \neq i \neq j \neq k$" is apparently used to mean "$l \notin \{i, j, k\}$."

O'Rourke himself explains one of the overarching problems of the book on p. ix of his preface: "It has become the fashion for textbooks to include far more material than can be covered in one semester. This is not such a text. It is a written record of what I cover with undergraduates in one 40 class-hour semester." The book does indeed have the feel of lecture notes. Terms are used before they are defined, and conditions relied upon before they are stated. While generalizations and failures to generalize (especially to higher dimensions) are mentioned, most details are omitted. And the many footnotes and digressions suggest places where students asked questions about definitions, notation, floating-point arithmetic, algorithmic complexity, topology, etc.

But lecture notes do not necessarily make good textbooks. An obvious reason is that every class is different. O'Rourke's students, for example, apparently understand without explanation topological terms like "open," "closed," and "bounded," but require explanations of cardinality, the floor function, and the relationship between logical conjunction and set intersection. I would expect most of my students to understand the latter but to need help with the former.

Lecture notes also impinge on the instructor's prerogatives, unlike more comprehensive textbooks that include more material than can be covered in one semester. A class that uses a textbook can explore various ways to define something or to pose a question, then turn to the text for a definitive rendering. When the text includes appropriately precise formulations, the instructor can explain in class that one of them means "let x wiggle." When the text includes complete verification of code correctness, the instructor can decide how many degenerate cases to check in class. In brief, lecture notes make it harder for the instructor to choose which issues to cover carefully, which to skim, and which to omit.

The preface suggests that the book is appropriate for undergraduates, graduate students, programmers, and those not interested in programming. Attempting to satisfy such a mixed audience, and simultaneously to cover a broad variety of topics, O'Rourke has opted for such superficial coverage that I wonder how much his audience can possibly understand about them. This is particularly noticable in Chapter 6, where duality and higher-order Voronoi diagrams receive only four pages each, and in Chapter 8, where "conceptual algorithms" abound. This would be a much better book if it devoted more attention to fewer topics, especially if those topics coincided better with O'Rourke's interests. The marked contrast between the author's obvious enthusiasm for clever mathematical ideas and his apologetic tone in explanations of code suggests that he might have done better to omit most of the programs altogether, while taking more care over the mathematical details.

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1. H. Edelsbrunner, *Algorithms in combinatorial geometry*, EATCS Monographs on Theoretical Computer Science, Springer, Berlin, 1987.
2. K. Mehlhorn, *Data structures and algorithms. Vol.* 3: *Multidimensional searching and computational geometry*, EATCS Monographs on Theoretical Computer Science, Springer, Berlin, 1984.
3. F. P. Preparata, *Computational geometry* : *an introduction*, Texts and Monographs in Computer Science, Springer, New York, 1985.

13[43–06, 43A32, 43A70, 42C15].—LARRY L. SCHUMAKER & GLENN WEBB (Editors), *Recent Advances in Wavelet Analysis*, Wavelet Analysis and Its Applications, Vol. 3, Academic Press, Boston, MA, 1994, xii + 364 pp., $23\frac{1}{2}$ cm. Price $59.95.

This is an important collection of papers by a truly distinguished group of contributors, and it should be in the hands of anyone who may be trying to keep abreast of the frantic activity in wavelet theory. All ten papers provide discussions in depth of the topics they address; they range in length from 22 to 62 pages.

In the first paper, Andersson, Hall, Jawerth, and Peters explore orthogonal and biorthogonal wavelets on prescribed subsets of the real line. A new concept of "wavelet probing" is introduced, and there are applications of wavelets to boundary value problems for ordinary differential equations. In the second paper, Auscher and Tchamitchian construct wavelet bases for elliptic differential operators in one dimension. These lead to estimates of the corresponding Green's kernel. Along the way, they create an extension of the multiresolution theory. In the third paper, Battle identifies the wavelets associated with Wilson's recursion formula in statistical mechanics. He then uses them to refine the formula. In the fourth paper, Benassi and Jaffard study a class of Gaussian random fields by means of wavelet decomposition. This work has applications in the theory of Brownian motion. In the fifth paper, Chui and Shi introduce multivariate wavelets in which the scaling (dilation) is different in each dimension, or, more generally, is defined by any nonsingular linear map. This innovation opens the possibility of constructing Riesz bases for multivariable functions using only a single function, together with the generalized notion of dilation and the usual translation by multi-integers. In the sixth paper, Dahmen, Prössdorf, and Schneider develop numerical schemes of the Petrov-Galerkin genre with trial spaces generated by a single scaling function. These schemes are applicable to a large class of pseudodifferential equations, and the authors discuss error estimates pertinent to that application. In the seventh paper, Daubechies discusses wavelet bases on an interval and the interaction of wavelets with the differentiation operator. At issue in the first topic is how to avoid edge effects when a wavelet basis on the line is restricted to an interval. In the second topic, the question is how to compute the derivative of a function efficiently from its wavelet decomposition. In the eighth paper, Donoho analyzes smooth wavelets whose dual bases consist of characteristic functions of intervals. He discusses the characterization of smoothness classes using such wavelet transforms. In the ninth paper, Flandrin and Gonçalvès study bilinear extensions of the wavelet transform. They explore the relation between time-scale and time-frequency energy distributions. In the final paper, Goodman, Micchelli, and Ward investigate spectral radius formulas for subdivision operators. They find related finite-rank operators with the same spectral radius. The rank of this associated operator depends only on the length of the mask in the subdivision operator and on the dimension of the underlying ambient space.

Each paper has a valuable bibliography, and the book has a subject index.

E. W. C.

14[28–00, 65D30].—DANIEL ZWILLINGER, *Handbook of Integration*, Jones and Bartlett, Boston, MA, 1992, xvi + 367 pp., $23\frac{1}{2}$ cm. Price \$49.95.

This book is a compilation of methods for dealing with integrals appearing in science and engineering problems. It starts with two introductory chapters, one on applications of integration and one containing concepts and definitions. This chapter closes with several sections on the transformation of integrals which is one of the more useful tools in evaluating integrals, both analytically and numerically. Chapter III discusses exact analytical methods, among them the use of computer packages which include a symbolic integrator. The next chapter on approximate analytical methods discusses among other techniques, asymptotic expansions, Laplace's method, stationary phase and steepest descent. The final two chapters are on numerical methods. Chapter V is concerned mainly with the use of Numerical Integration Software while Chapter VI discusses some of the standard numerical integration techniques such as adaptive integration, Clenshaw-Curtis and Gauss-Kronrod rules, cubic splines, lattice rules, Monte Carlo and number-theoretic methods, etc. Each section contains a particular procedure, usually followed by an example, notes and references. The notes are important for the understanding of the main text and sometimes correct inaccuracies therein. They also extend the scope of the text and provide many of the references. The references range from the standard sources to the recent literature.

This book is very uneven. On the one hand, it contains many sections of substance and great practical interest; on the other hand, it contains much trivial and useless material. Thus, the section on the MIT Integration Bee is entirely superfluous, nor could I see much point in the section on integral inequalities, even though it gave an impressive list of such inequalities. The section of excerpts from GAMS could be dispensed with and replaced with a reference and similarly with the collection of integration formulas over planar regions. In place of these sections, I would have liked to see a treatment of Sinc rules for one-dimensional integration and a discussion of periodization in quasi-Monte Carlo rules for multidimensional integration.

There are more than the usual quota of typographical errors and many questionable statements as well as much material which could use further elaboration. However, in spite of its shortcomings, the book serves a useful purpose and should be of benefit to the audience to whom it is addressed.

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15[65–00, 65–02, 65D30].—H. V. SMITH, *Numerical Methods of Integration*, Studentlitteratur, Chartwell-Bratt, Kent, England, 1993, iv + 147 pp., $22\frac{1}{2}$ cm. Price: Softcover $17.00.

This reference monograph summarizes a class of one-dimensional quadrature methods. Typically, the results given are presented in the form

$$\int_a^b w(x)f(x)\,dx = \sum_{j=1}^{n} w_j f(x_j) + E_n(f),$$

along with explicit descriptions for the weights, w_j, the abscissae, x_j, and the error term, $E_n(f)$. In addition, the author often adds brief comments on the evaluation of the sum approximating the integral, or on the error term. Although derivations of the formulae are omitted in the monograph, references are given to paper publications where the formulae are derived and discussed in detail. In addition, one finds worked examples illustrating the application of the formulae, as well as supplementary problems at the end of each chapter.

The monograph is divided into the following chapters:

1. Newton-Cotes Quadrature
2. Gauss-Type Quadrature Rules
3. Chebyshev Polynomials
4. The Error Term
5. Kronrod Quadrature
6. Oscillatory/Periodic Integrals
7. Integrals Involving Singularities
8. Infinite, Semi-Infinite Integrals
9. Divergent Integrals

These titles are sufficiently descriptive to give the reader an idea of the content of each chapter.

At the end of the monograph, one also finds several pages devoted to each of the following:

(i) Solution to Selected Supplementary Problems
(ii) Appendix A. NAG
(iii) Appendix B. Tables
(iv) Bibliography
(v) Index

The above headings (i), (iv) and (v) are sufficiently descriptive to make their content self-explanatory. The Appendix A, NAG, contains a brief reference to the use of the quadrature routines in the NAG (Numerical Analysis Group Project) Library, which was developed collectively by five British universities.

The tables in Appendix B are tables of numbers which may be used to obtain global error bounds, as explained in the monograph, for Gauss-Legendre, Gauss-Chebyshev, Newton-Cotes, Lobatto, and Radau quadrature.

The text is mainly a reference text, even though it contains some good problems which weigh its purpose towards the direction of instruction.

F.S.

16[41A55, 65D30, 65D32].—H. Brass & G. Hämmerlin (Editors), *Numerical Integration* IV, International Series of Numerical Mathematics, Vol. 112, Birkhäuser, Basel, 1993, xii + 382 pp., 24 cm. Price $100.50.

This is the fourth volume of Proceedings of Conferences on Numerical Integration in which Professor Hämmerlin has been either the Editor or Coeditor. The Proceedings of the previous three Conferences have also been published in this International Series of Numerical Mathematics (see Vols. 45, 47 and 85). Of these four volumes this latest one is by far and away the best produced and bound; the publishers, Birkhäuser, have done an excellent job. In this volume we find 27 refereed papers (see Contents, below) together with an addendum containing nine unsolved problems. This Conference, held at the Oberwolfach Mathematics Research Institute, was attended by 46 mathematicians from 16 different countries with at least one delegate from each of the 5 Continents.

To misquote slightly from the Preface to the Third Edition of Gabor Szegö's book on Orthogonal Polynomials, "The interest of the mathematical community for numerical integration is still not entirely exhausted". This is perhaps surprising in the light of the Editors' statement that "Algorithms for the numerical computation of definite integrals have been proposed for more than 300 years, ... ". However, the Editors go on to say that ... "practical considerations have led to problems of ever increasing complexity so that, even with current computing speeds, numerical integration may be a difficult task. High dimension and complicated structure of the region of integration and singularities of the integrand are the main sources of difficulties". Of the 27 papers, 17 were on one-dimensional, and 10 on multivariate approximate integration. Of the papers on one-dimensional quadrature nearly all of them (13) relate to either orthogonal polynomials or Gauss quadrature. It seems that orthogonal polynomials are in great demand in numerical integration; Gabor Szegö would, I am sure, be pleased.

To underline the continuing interest in numerical integration, this volume concludes with a report of one of the evenings of the Conference at which open problems were discussed. A total of nine problems is listed, eight of them concerning one-dimensional quadrature rules. In the tradition of Paul Erdös, it is stated that Frank Stenger is offering a 50 DM reward for a solution to the problem he proposed. (An e-mail to Frank on 26th April 1994 elicited the response that his problem remains unsolved and so keen is he to have it resolved one way or the other that he is now offering US$100 in place of the original DM50.)

Following the Oberwolfach tradition, all the papers, with one exception, are in English. This volume is an excellent addition to the ISNM series and any one who has the least interest in quadrature should make sure that they have ready access to it.

CONTENTS

The volume concludes with a section containing nine unsolved problems.

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17[60–02, 60G07, 60H10, 65C05, 65C20].—NICOLAS BOULEAU & DOMINIQUE LÉPINGLE, *Numerical Methods for Stochastic Processes*, Wiley Series in Probability and Mathematical Statistics, Wiley, New York, 1994, xx + 359 pp., 24 cm. Price $64.95.

This book offers a rigorous exposition of numerical treatments of stochastic models. A considerable mathematical sophistication is expected of the reader, but a brief review of the prerequisites is provided in the first chapter. The authors distinguish two types of simulation methods, the Monte Carlo method based on the strong law of large numbers, and the shift method based on the pointwise ergodic theorem. The shift method is particularly appropriate in infinite-dimensional settings.

Chapter 2 describes the mathematical framework for the Monte Carlo method. There is also some material on quasi-Monte Carlo methods, but here more extensive and up-to-date treatments are available in other sources, e.g., in the CBMS-NSF monograph of the reviewer [1]. Chapter 3 discusses the simulation of random processes and random fields in an infinite-dimensional setting. Markov processes, point processes, and processes with stationary independent increments are highlighted. Chapter 4 deals with the deterministic resolution of some Markovian problems through methods such as balayage algorithms and the reduced function algorithm. The carré du champ operator is applied to hedging strategies in financial markets. The last chapter is devoted to the numerical resolution of stochastic differential equations and the computation of expectations of random variables defined on Wiener spaces.

The book is on the whole very reliable and accurate. There are only some minor quibbles, for instance, the title of the paper of Warnock (1972) is given incorrectly. Readers seeking an introduction to the area will find the style of the book somewhat terse.

H. N.

1. H. Niederreiter, *Random number generation and quasi-Monte Carlo methods*, SIAM, Philadelphia, PA, 1992.

18[68Q40, 65Y25, 11–04, 12–04, 13–04, 14–04, 30–04, 33–04].—JOHN GRAY, *Mastering Mathematica: Programming Methods and Applications*, AP Professional, Boston, 1994, xx + 644 pp., $23\frac{1}{2}$ cm. Price: Softcover $44.95.

If you are a mathematician familiar with, or interested in, the *Mathematica* programming system and if you share even some of the author's eclectic set of interests, you may find this book useful.

Apparently intended for a course in mathematical software, this book's orientation—overwhelmingly one of endorsing Mathematica as the answer, regardless of the question—seems inappropriate as sole text for such a course. It may be viable as additional "symbolic methods" reading in combination with a numerical methods text.

Part I (Using Mathematica as a Symbolic Pocket Calculator), 140 pages, and Part II (Mastering Mathematica as a Programming Language), 260 pages, fits somewhere between Blachman's introductory book [2] and Maeder's book on advanced programming [3]. Gray's introduction to selected parts of the system is not entirely authoritative (there are even occasional typos in the computer-generated figures) but may be just right for an audience of upper-division applied mathematics students.

Part III in 110 pages illustrates computing in some areas of group theory and differentiable mappings of particular interest to the author. The last 110 pages are answers to problems.

If you wish to learn about ideas of programming languages partly covered in Part II: functional programming, object-oriented programming, the use of a few ideas from lambda calculus, etc., you may find (for example) the text by Abelson et al. [1] far more complete and authoritative than the coverage here.

Re-interpreting such ideas in a *Mathematica* framework has a number of failings, one of which is that it sometimes "reduces" simple ideas to complicated ones; another is that the "implementation" is extremely inefficient in execution time. However, a reader who would like to understand how something might be computed by relating it to an implementation in *Mathematica* may find the systematic development of such ideas as object-oriented programs of some interest.

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1. H. Abelson, G. J. Sussman, and J. Sussman, *Structure and interpretation of computer programs*, MIT Press, Cambridge, MA, 1985.
2. N. Blachman, *Mathematica, a practical approach*, Prentice-Hall, Englewood Cliffs, NJ, 1992.
3. R. E. Maeder, *Programming in mathematica*, 2nd ed., Addison-Wesley, Reading, MA, 1991.

19[14–06, 13–06, 13P10, 14Qxx].—DAVID EISENBUD & LORENZO ROBBIANO (Editors), *Computational Algebraic Geometry and Commutative Algebra*, Istituto Nazionale di Alta Matematica Francesco Severi, Symposia Mathematica, Vol. 34, Cambridge Univ. Press, Cambridge, MA, 1993, x + 298 pp., $23\frac{1}{2}$ cm. Price $49.95.

This small, attractively bound volume consists of a collection of papers from a conference on the topics in its title held in 1991 in Cortona, Italy. Most of the papers deal with the theory of Gröbner bases, although there are some interesting exceptions. The papers are, as a group, of very high quality, and several of them are first-class contributions to the expository literature on this important and interesting subject. Before discussing some of the papers in more detail, we will briefly consider the role of the Gröbner basis algorithm.

The Gröbner basis algorithm is a generalization of the Euclidean algorithm to systems of polynomial equations in many variables. Using it, one may carry out explicitly many of the fundamental operations in commutative algebra, such as deciding if one polynomial belongs to the ideal generated by a finite list of others, or eliminating variables from a system of polynomial equations. More generally, the algorithm makes effective the process of determining syzygies—algebraic relations—among systems of polynomials. Since many questions in commutative algebra and algebraic geometry eventually boil down to problems involving syzygies, the Gröbner basis algorithm is extremely useful in attacking computational problems in these fields.

One of the more attractive features of the theory of the Gröbner basis algorithm is its inherently interdisciplinary nature. The algorithm is of interest to algebraists and algebraic geometers who wish to carry out explicit computations to answer questions in their "pure" research and to computer scientists interested in complexity bounds. These two groups are driven together because it has become clear that the behavior of the Gröbner basis algorithm applied to a system of polynomials is determined by geometry of the algebraic variety (or scheme) defined by that system. Computer scientists interested in the complexity of the algorithm must understand the geometric significance of that complexity, while algebraic geometers who want to be able to carry out a particular calculation in a reasonable amount of time must understand the significance of the complexity results. Both the complexity theory and practical geometric applications of the Gröbner basis algorithm are addressed in the volume under review, although the geometric applications receive more emphasis.

We turn now to the articles in this volume. The first two articles, entitled "What can be computed in algebraic geometry?" by Dave Bayer and David Mumford, and "Open problems in computational algebraic geometry" by David Eisenbud, are beautiful presentations of the central issues in the field. They should become standard references in this area. The Bayer and Mumford article presents a very clear yet sophisticated introduction to the theory of Gröbner bases, and discusses in detail the relationship between the *regularity* of an ideal, a cohomological measure of the complexity of the ideal, and the performance of the Gröbner basis algorithm. They also present examples of various kinds of worst-case performance, keeping in mind the relationship between geometry and complexity. Finally, they discuss, in general terms, some applications of the algorithm.

Eisenbud's article presents a series of open problems. Some of these problems fall into the general picture susceptible to attack by the Gröbner basis algorithm, but others clearly do not. Some of the problems he discusses include resolving surface singularities and making the classification of surfaces effective; others of a different flavor include finding rational points on varieties, which is clearly of a very different, non-Gröbner character. For the geometer, this article helps to make clear what is meant by "computational" algebraic geometry, since it sharply points out the differences between what can be done "in theory" and what can be done explicitly.

The remainder of the articles in the volume are more specialized. Papers by D. Lazard ("Systems of algebraic equations: algorithms and complexity") and by

T. Mora and L. Robbiano ("Points in affine and projective spaces") consider the theory of zero-dimensional varieties from two points of view. Lazard describes various approaches to finding the solutions to a system of polynomial equations whose common zeros are a finite set of points; in this setting there are a number of algorithms more or less closely related to the Gröbner basis method. Mora and Robbiano consider the opposite problem of finding the ideal of polynomials which vanish on a specified set of points or zero-dimensional subscheme of projective space.

A long article by W. Vasconcelos ("Constructions in Commutative Algebra") surveys methods for solving certain explicit problems in algebra, such as computing integral closure and primary decomposition. The article applies homological techniques (i.e., syzygies) as much as possible to these problems. To take advantage in practice of the methods discussed in this article, the reader should be familiar with algorithms for more elementary constructions, such as computing $(I : J)$ for ideals I and J.

In his article "Sparse elimination theory," Bernd Sturmfels discusses some of the connections of the Gröbner basis algorithm with the combinatorial theory of polytopes and (implicitly) toric varieties. This article seems somewhat out of context in this volume, but in fact is an important signpost to a fascinating related area of beautiful mathematics.

We will briefly mention the other articles in the volume. D. Bayer, A. Galligo, and M. Stillman, present an analysis of the behavior of Gröbner bases under base extension ("Gröbner bases and extensions of scalars"), which among other things provides a very concrete interpretation of the concepts of "flatness" and "faithful flatness." A paper by M. Giusti and J. Heintz ("La détermination des points isolés et de la dimension d'une variété algébrique peut se faire en temps polynomial") analyzes the problem in its title in the spirit of complexity theory. Sheldon Katz shows how Macaulay, a system which actually carries out the Gröbner basis algorithm and computes sheaf cohomology, can be used in practice to attack a problem in geometry ("Arithmetically Cohen-Macaulay curves cut out by quadrics"). Finally, Th. Dana-Picard and M. Schaps, in the only paper in the volume which does not at least mention Gröbner bases ("A computer assisted project: classification of algebras"), consider the problem of classifying finite-dimensional algebras by homological methods.

Physically, this volume has a professional quality binding and the papers were prepared in a reasonably consistent dialect of TEX.

In summary, this is a compact conference proceedings volume containing generally high-quality papers and two excellent expository articles on computational algebraic geometry and commutative algebra.

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