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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

38[65-01, 65Bxx, 65Cxx, 65Dxx, 65Fxx, 65Gxx]—*Numerical analysis: A first course in scientific computation*, by Peter Deuflhard and Andreas Hohmann, (translated from the German by F. A. Potra and F. Schulz), Walter de Gruyter, Berlin, 1995, xiv+355 pp., 23 cm, hardcover \$69.95, paperback \$39.95

The authors present pseudocodes which clearly define algorithms that solve many of the basic types of problems arising in Scientific Computing today. The mathematical explanation of the numerical methods is well motivated, precise, and carefully considers the notions of condition and stability. Well-drawn figures exhibit the geometric interpretation of the mathematical arguments used in the areas of Bézier approximation, bifurcation, B-splines, continuation, orthogonality, stability, etc. The reader is assumed to have "basic knowledge of undergraduate Linear Algebra and Calculus". This reviewer believes that mathematical maturity on at least the upper level undergraduate or beginning graduate level is required to master material of this depth. The English translation is an excellent mathematical presentation.

The reviewer highly recommends the work as a textbook and as a reference resource for all. The material is organized into nine chapters and is ample for a fine full-year course. The authors indicate various ways to incorporate some of the chapters into special topics courses. The book leads up to but does not discuss numerical methods for solving differential equations.

Each chapter contains pseudocodes, illustrative numerical examples (some taken from the engineering literature), and concludes with a set of well-chosen exercises for the reader. The authors state that all of the algorithms mentioned in the text are freely available via the Internet and give directions for accessing them at the electronic library, eLib, of the Konrad Zuse Center.

The title, number of exercises (ex), and number of pages (pg) is listed for each chapter in the Contents:

Eugene Isaacson

39[65N30, 65N50, 65N55, 82D99]—*Multigrid methods for semiconductor device simulation*, by J. Molenaar, CWI Tract, Vol. 100, Centre for Mathematics and Computer Science, Amsterdam, 1993, vi+134 pp., 24 cm, softcover, Dfl. 40.00

In recent years, multigrid methods have found their way into a number of important application areas. Often it seems that the careful analysis which would lead to a useful evaluation of multigrid as a viable method for such problems is ignored or lost along the way. However, from time to time an article or book appears which studies a specific application in depth, and also examines in detail the theoretical and computational properties of the multigrid method for the particular problem. The book is an example of such a study.

The book, based on the author's PhD thesis, is concise and well organized, consisting of seven chapters and two appendices. The material begins in Chapter 1 with an overview of the three main approaches to device simulation, followed by a more detailed description of one of these approaches, namely the numerical solution of the drift-diffusion equations proposed by Van Roosbroeck in the 1950s. The drift-diffusion equations are examined carefully, including the scaling problems, which lead to the use of various formulations as alternatives to the Slotboom variable representation. Basic discretization issues for the drift-diffusion model are also presented, including a discussion of the importance of the well-known Scharfetter-Gummel discretizations, leading into a discussion of the mixed finite element approach (the subject of Chapter 2). The introductory chapter ends with a quick look at methods for the resulting discrete equations (multigrid, Newton-type methods, etc.), and with a detailed outline of the remainder of the book.

Chapter 2 consists of a careful discussion of mixed finite element discretization of the semiconductor drift-diffusion equations, in both one and two dimensions. The motivation for the use of a dual mixed finite element approach is that it can be viewed as a mechanism for extending the Scharfetter-Gummel scheme to more than one dimension, at the same time having available the complete analysis framework that the finite element method provides for an examination of the error. A discretization of a general model elliptic equation is derived in two dimensions on a rectangular mesh. However, as has been shown in other contexts [1], such a discretization is not stable in the sense that an M-matrix is not obtained (a discrete maximum principle is then not available). However, the author obtains an M-matrix through the use of mass-lumping, and his analysis shows that the quadrature rule this corresponds to does not spoil the accuracy of the discretization. (The author does not make it clear that such an approach will not work in three dimensions, since an M-matrix cannot be recovered simply by mass lumping [1].) Applying the discretization to drift-diffusion equations yields the sought-after two-dimensional extension of the Scharfetter-Gummel scheme.

Chapter 3, one of the most interesting chapters of the book, takes a close look first at abstract optimization problems, and then more specifically at special relaxation methods (which find use later in the book as multigrid smoothers). The weak-form equations used in Chapter 2 are reformulated as variational problems in two different ways, yielding constrained optimization problems. Taking this dual view of nonlinear problems is a common technique in developing robust solution algorithms for the weak-form equations. This technique consists of (1) selecting a minimization problem (either the natural one in the Euler-Lagrange sense, or simply the norm of the residual of the weak equations) such that the solution to the minimization problem also solves the weak-form equations, and (2) constructing a solution algorithm for the weak-form equations which is guaranteed to decrease the value of the "cost" function of the associated minimization problem. Convergence of the method thus constructed is then assured. The author considers two relaxation methods (superbox and Vanka-type) for the weak-form equations, and shows that each minimizes an appropriate associated functional, and hence convergence is guaranteed. The chapter finishes with a local mode analysis of each relaxation method in an attempt to evaluate the possible effectiveness of each as a multigrid smoother.

Chapters 4 and 5 consider two- and multigrid methods based on the two relaxation methods of Chapter 3, for a mixed finite element discretization of the Poisson equation. A standard two-grid analysis is performed, yielding bounds on the spectral radii and norm of the error propagator. It is shown that while the "canonical" grid transfer operators may cause a problem in the one-dimensional case for certain orderings in the relaxation, no such problem occurs for the two relaxation methods in two dimensions. While a similar multigrid analysis is not possible, the author describes in detail a nonlinear multigrid algorithm for the full nonlinear drift-diffusion model. He focuses mainly on Gummel's method as an outer nonlinear iteration, and solving the individual equations in each Gummel iteration with multigrid. Many numerical results are presented, illustrating the effectiveness of the two relaxation methods, as well as the overall multigrid method. A claim is made that employing simply (an inexact) Newton's method and possibly a linear multigrid method for the Jacobian systems would not be as effective, owing to the likely ill-conditioning of the Jacobians. However, no comparisons are made numerically, and it is not clear that such a Newton/linear multigrid combination would not be equally effective (see for example [2]).

Chapter 6 considers the use of adaptive mesh refinement with the methods presented in Chapter 5, and a particular refinement criterion is proposed. Some numerical experiments comparing such an adaptive implementation with a uniform mesh implementation show convincingly the utility of adaptive mesh refinement for this problem. The book finishes in Chapter 7 with a careful examination of two different multigrid discretization frameworks, based on two different mixed finite element approaches. It is shown that each is equivalent to a particular box (finite volume) discretization, and leads to either the so-called cell-centered or vertex-centered multigrid methods. It is concluded after some analysis that the vertex-centered approach is both more robust and has faster convergence properties than the cell-centered approach. Two appendices contain some descriptions of various implementation issues which arose in the work.

This book is clearly written and contains few typographical errors. It could be used in a course on numerical semiconductor modeling, or in a course on advanced multigrid techniques for nonlinear elliptic systems.

References

1. T. Kerkhoven, *Piecewise linear Petrov-Galerkin error estimates for the box-method*, SIAM J. Numer. Anal. (1997) (to appear).
2. T. Kerkhoven and Y. Saad, *On acceleration methods for coupled nonlinear elliptic systems*, Numer. Math. **60** (1992), 525–548. MR **92j:**65084

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40[73-06, 73K05, 73K10, 73K15, 73V25]—*Asymptotic methods for elastic structures*, Philippe G. Ciarlet, Luís Trabucho, and Juan M. Viaño (Editors), de Gruyter, New York, 1995, viii+297 pp., $24\frac{1}{2}$ cm, $128.95

This book is the proceedings of the international conference on "Asymptotic Methods for Elastic Structures" held October 4–8, 1993 in Estoril, Portugal. Twenty-one of the twenty-three speakers at the conference contributed papers to this volume, most of which are between ten and fifteen pages in length. The papers deal with a variety of topics in the theory of beams, plates, rods, shells, and their assemblages. The unifying theme is that all these models are lower-dimensional approximations to higher-dimensional elastic structures which have a small thickness. Some of the topics considered are numerical approximation of the models, existence and uniqueness results, controllability, convergence and error estimation between the original and reduced model, the modelling of problems with junctions, and derivation and justification of models by asymptotic expansions.

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41[65-00, 65-04, 41-00, 41-04, 41A15]—*Handbook on splines for the user*, by Eugene V. Shikin and Alexander I. Plis, CRC Press, Boca Raton, FL, 1995, xii+221 pp., 24 cm, $69.95

According to the authors, this book is intended as a handbook for prospective and active spline users. It is not a textbook, it does not provide any proofs, only states results, and it is limited to the description of cubic splines techniques in one and two dimensions, and their implementations, including a set of programs on diskette.

The book consists of four chapters, each one designed to be read independently of the others. Chapter 1 deals with univariate cubic splines for interpolation (with various end conditions) and smoothing. In Chapter 2, the corresponding tensor-product versions, i.e., interpolating and smoothing bicubic splines, are presented. Spline curves are the topics of Chapter 3. After a short introduction of basic curve theory, there are subsections on cubic Bézier curves, B-spline curves, Beta-splines and one on other approaches such as Hermite, Catmull-Rom and implicitly defined spline curves. Finally, spline surfaces are described in Chapter 4. Again, after some introductory material on surfaces, there are subsections on Bézier surfaces, B-spline surfaces and Beta-spline surfaces, and the final one on Hermite and implicit cubic spline surfaces. Each subsection ends with comments on the implementation of the presented methods in the enclosed set of programs called Spline Guide, designed for an IBM PC or compatible machines. The interpolation and smoothing programs for the first two chapters are written in Fortran, the programs for the curves and surfaces of Chapters 3 and 4 are coded in C.

There are no references at the end of each subsection or chapter, only one list at the end of the book, which is very short and leaves out quite a number of pertinent books on splines. Therefore, in my opinion, there is no help for an uninitiated reader to find out about other splines, for example quadratic ones, or about proofs, or even who introduced the concepts and algorithms, which would assist in further studies. The reader is supposed to be content with the material as it is presented in the book and implemented in the programs. Finally, there are instances where the usage of English in the text should have been more carefully monitored by the publisher.

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42[65-01, 65C05, 65C10]—*A primer for the Monte Carlo method*, by Ilya M. Sobol′, CRC Press, Boca Raton, FL, 1994, xx+107 pp., $21\frac{1}{2}$ cm, softcover, $41.95

This is the English translation of a book which first appeared in Russian in 1968 and for which earlier translations into English have been brought out, e.g. by Mir Publishers, Moscow, under the title "The Monte Carlo Method". The book is still useful as a brief introduction to the subject for students. However, the bibliography is rather poor since it contains only 10 items.

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43[65-01, 68-01, 68U05]—*An introduction to scientific, symbolic, and graphical computation*, by Eugene Fiume, A K Peters, Wellesley, MA, 1995, xvi+306 pp., $23\frac{1}{2}$ cm, $49.95

This undergraduate textbook is positioned at the interface between computer science and mathematics. It can be seen as an applied mathematics course with the field of application being computer graphics. Computer science students will get a useful introduction to the nontrivial mathematical prerequisites of computer graphics, and mathematics students will be exposed to an interesting field of application that is not taken from science or engineering. The topics are widely applicable in many fields, and this justifies the general title of the book.

A theme of the book is that symbolic and graphical uses of computers have become more or less equal partners with strictly numerical processing in developing solutions of scientific and other kinds of problems. These seminumerical uses have been present alongside numerical computation since the beginning of digital electronic computing but their impact has been diminished until recently by the lack of affordable and sufficiently powerful interactive systems. Now such systems as Maple, Mathematica and Matlab are installed routinely on personal computers and workstations.

The book employs Maple to introduce students to this enlarged view of scientific computing (which has already been adopted by much of the professional community of scientific computer users). Many examples and exercises are presented in terms of Maple sessions and procedures. A much smaller number are presented as programs in Turing or C. In comparison to the latter two languages, Maple facilitates the rapid generation of mathematical developments, both symbolically and numerically, together with ready access to graphical displays. This is particularly appropriate in a course that delivers a steady flow of applicable mathematical concepts.

The undergraduate character of the book is indicated by its avoidance of linear algebra and multivariate calculus. The author states that "single-variable calculus is the only real prerequisite (or corequisite), although some programming experience would be helpful." Unsurprisingly, there are no proofs. The descriptions, derivations and plausible arguments are all quite convincing.

The book contains the following chapters.

Chapter 0, Mathematical Computation, provides background material and illustrative examples of symbolic, graphical and numerical computation.

Chapter 1, Representation of Functions, discusses explicit, implicit and parametric representations with emphasis on polynomials. Examples include plane curves, space curves, and geometric figures such as the Koch snowflake. A good introduction to line and circle rendering on a raster graphics device includes the step-by-step development of an accurate and efficient integerized algorithm.

Chapter 2, Interpolation, is motivated by the space curve traced by the flight of a fly and sampled at discrete points by a "Fly Tracker." The emphasis is on piecewise polynomial interpolation with each polynomial parameterized for evaluation on the domain $[0, 1]$. Accordingly, reparameterization and change-of-basis are discussed in some detail. The cubic Catmull-Rom interpolant (important in computer graphics and related to the cubic Hermite interpolant) is developed and compared favorably to Lagrange interpolants because of its smooth transition between curve segments.

Chapter 3, Approximation and Sampling, contains two parts. In the first part interpolation is compared to approximation by considering cubic Catmull-Rom and cubic B-spline bases. In the second part the subject of signal processing is introduced. Filters and their connection via convolution to Fourier series and integrals are discussed with many illustrative examples. The chapter concludes with a description of the sampling theorem.

Chapter 4, Computational Integration, derives quadrature rules based on the midpoint, trapezoid, Simpson and Catmull-Rom formulas. Their performance is compared numerically on four simple integrals (including an arc length). The chapter concludes with a discussion of Monte Carlo integration.

Chapter 5, Series Approximations, introduces the floating-point representation and rounding error of arithmetic operations. Then Taylor expansions are introduced and used to derive the familiar truncation error formulas for the quadrature rules presented in Chapter 4. Finally, the Fourier inversion formulas and the sampling theorem are revisited for further discussion.

The subject of the final Chapter 6, Zeros of a Function, is motivated by intersection problems in geometric modelling. Both symbolic and numerical approaches are discussed.

From the foregoing description it can be seen that this textbook covers a wealth of material from a somewhat unusual viewpoint. A number of misprints, mostly minor, were detected. Some of the mathematical notation seems excessive, or excessively pedantic, but not more than is usually observed in texts at this level. The index failed to provide the needed page reference on a few occasions when it was consulted.

The author maintains a World Wide Web site to provide services for and obtain feedback from users. Among the services provided are programs and procedures related to topics in the book, and an errata sheet.

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44[65-02, 65Yxx]—*High performance computing—Problem solving with parallel and vector architectures*, Gary W. Sabot (Editor), Addison-Wesley, Reading, MA, 1995, xvi+246 pp., 24 cm, $45.14

This book attempts, by means of a number of case studies from scientific computing, to illustrate the techniques for developing efficient programs on high-performance computers. Applications from shock-wave physics and weather prediction have been included. These applications, as well as algorithms from numerical linear algebra, dynamic tree searching, graph theory, and mathematical programming have been implemented in several programming languages, and parallel architectures and their performance analyzed and reported. The chapters of the book correspond to contributed case studies whose intent is to identify and address high-performance computing issues at the application, algorithm, language, and machine levels. The issues of portability and scalability are addressed in the last chapter of the book and within each case study. The spectrum of target machines covered ranges from SIMD and MIMD massively parallel machines to vector machines and network of workstations. The parallelization techniques and methodologies presented are tightly coupled with the particular case studies selected. The material of the book could be useful to application software developers and could provide supplementary topics for an introductory graduate course in high-performance computing.

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45[65F05, 65F15, 65F20, 65F35, 65Y05, 65Y20, 65Y99]—*LAPACK Users' guide*, by E. Anderson, Z. Bai, C. Bischof, J. Demmel, J. Dongarra, J. du Croz, A. Greenbaum, S. Hammarling, A. McKenney, S. Ostrouchov, and D. Sorensen, second edition, SIAM, Philadelphia, PA, 1995, xx+325 pp., 28 cm, softcover, $28.50

As a successor to the software packages LINPACK and EISPACK, LAPACK provides more efficient and accurate routines for the solutions of dense systems of linear equations, least squares problems, eigenvalue and singular value problems. The second edition of the User's Guide is mainly for the September 30, 1994 release of version 2.0 of the package. In addition to what the first edition offers, the new edition provides pointers to the guides of several related packages including

LAPACK++ (C++ extension to LAPACK), CLAPACK (a C version of the entire LAPACK obtained by using the automatic Fortran to C conversion program), ScaLAPACK (a subset of LAPACK routines that run on certain distributed memory parallel computers), and some routines exploiting IEEE arithmetic. A number of algorithms are added to the new release of LAPACK and further discussed in the User's Guide. They are for the generalized nonsymmetric eigenproblems, the generalized linear least squares problems, the generalized singular value decomposition and a generalized banded symmetric definite eigenproblem. There are also new routines that implement the divide-and-conquer methods for symmetric eigenproblems.

Chapter 1 contains a brief introduction to LAPACK and its availability, including the World Wide Web URL address. It also provides references for several related packages mentioned earlier. Discussions on how to use CLAPACK and the difference in the definition, as well as memory allocation, of a two-dimensional array in Fortran and C are included in Section 1.11.2. I think this is useful for any C users of the package. There is no discussion on efficiency issues related to the conversion from Fortran to C.

Chapter 2 provides the contents of LAPACK and a short introduction for each algorithm. A number of subsections are added to describe the newly implemented routines.

Chapter 3 presents some performance measurements for LAPACK. Some of the old computers used in obtaining performance figures in the first edition of the User's Guide are replaced by newer machines. A new section, LAPACK Benchmark, is added to this chapter. It contains performance numbers for some of the most commonly used routines in numerical linear algebra on a variety of workstations, vector computers, and shared memory parallel computers.

Chapter 4 discusses accuracy and stability issues. The presentation is different from what appeared in the first edition and, in my opinion, is easier to read. Much of the detailed theory is discussed in separate sections marked with "Further Details". A few Fortran code fragments are included in this chapter to calculate the errors for certain quantities computed by LAPACK. These code fragments are useful, at least for expert users.

The rest of the book includes Chapter 5 for documentation and convention, Chapter 6 for installation, Chapter 7 for troubleshooting and Appendices A, B, C, D, E for indices, reference to BLAS, converting from LINPACK or EISPACK and a list of LAPACK Working Notes. Part 2 contains specifications of all the routines.

Overall, this is a well-written book, and I highly recommend it to those who are new to LAPACK or have used LAPACK and are interested in understanding or using the new routines released in version 2.0 of LAPACK.

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46[00A20, 68Q40]—*The Maple V handbook*, by Martha L. Abell and James P. Braselton, AP Professional, Boston, MA, 1994, viii+726 pp., $23\frac{1}{2}$ cm, softcover, $39.95

The authors describe this book as "a reference book for all users of Maple V, in particular, for students, instructors, engineers, business people, and other professionals who use Maple V in their work" (p. vii). They aim to satisfy users who are looking for a reference "between the very elementary handbooks available on Maple V and those reference books written for advanced Maple users" (p. vii). This is a worthy, and challenging, objective. The key element of such a book should be an up-to-date, accurate, and comprehensive summary of all commands available in Maple, organized in an accessible format. Measured against these standards, the final product unfortunately falls short of the authors' objectives.

The original introductory Maple reference is *First Leaves: A Tutorial Introduction to Maple V* [5]; other books that attempt to achieve the same goals as *The Maple V Handbook* include *Maple V Quick Reference* [2]*, Introduction to Maple* [6] and *The Maple Handbook* [7]. I will evaluate the current book both against the criteria listed above (ease of use and content), and relative to its prime competitors ([2], [5], [6] and [7]).

The Maple V Handbook begins with a one-chapter (23 pages) introduction to Release 2 of Maple V. The discussion includes information for users of a one-chapter (23 pages) introduction to Release 2 of Maple V. The discussion includes information for users of the Macintosh, Windows, DOS, and UNIX implementations of the Maple worksheet environment and an introduction to the Maple on-line help system. There is virtually no introduction to Maple itself (e.g., assignments vs. equations, default data type assumptions, and evaluation rules). Even experienced users often need to be reminded of these fundamental principles. By comparison, [5], written for Release 1, provides an extensive and accessible introduction (at the novice level). Both [2] and [7] have been updated for Release 3; each contains an overview of the Maple environment that is appropriate for intermediate users. Although written for Release 2, the introductory chapters of [6] provide both an excellent introduction and additional technical information needed by intermediate and advanced users.

The introduction to the Maple on-line help provided in *The Maple V Handbook* is effective, but slightly dated because keyword searches were not supported in the worksheet environment in Release 2. While this section should be improved, it is surpassed only by [2] and [6].

The remaining sixteen chapters of the book describe the built-in functions (Chapter 2, 192 pages), miscellaneous library functions (Chapter 3, 40 pages), and fourteen packages that are provided with Maple (Chapters 4–17, 458 pages). Each chapter begins with a brief explanation of the types of commands contained in the chapter, including general `help` information and any special steps needed to load the appropriate additions to Maple. The organization is most similar to [2]; [5] and [6] make no attempt to be comprehensive. The most flexible organization, commands grouped by general subject area and extensively cross-referenced, is found in [7].

The organization of material can substantially affect the ease with which information can be extracted. In this area *The Maple V Handbook* is disappointing. For example, suppose a user wants to solve a differential equation, but does not know the appropriate Maple command. The index contains only Maple command names and keywords. Thus, there are no entries for "differential", "equation", or other related keywords. The closest matches seem to be `Diff` and `diff`. Looking through these entries, in order, we encounter an example for the `assign` command

(p. 48) which includes finding the solution to an initial value problem. From this information one infers that `dsolve` is a Maple command for solving differential equations. Other paths to this same information include guessing that the `DEtools` package (Chapter 5) has something to do with differential equations or examining the list of cross references listed under the `solve` command. Each of these paths is indirect, and inefficient. In comparison, a direct path to `dsolve` is easily found in each of [2], [5], [6] and [7]; the keyword searches performed from the command line (by `help` or `?`) or from the worksheet are even more efficient. In general, the index in *The Maple V Handbook* seems to be effective only when the name of the Maple command or keyword is known in advance.

Beyond the question of organization, what information is provided? Each entry contains, in order, a list of cross references, a brief explanation of the basic command syntax, and one or more examples illustrating how the command might be used. The cross references are extensive, but often refer to information not contained in this handbook. The explanations are correct, but sometimes incomplete. Attention to data types is essential for the effective use of Maple; this book is very inconsistent on the matter. In contrast, the information in [7] is more detailed and technical, but contains no examples; the cross references are generally excellent, including explicit references to [3], [4], and [5]. The entries in [2] are technically complete, easy to read, and concise. There are, however, only limited examples and no Maple output. In contrast, [6] presents a large number of illustrative examples and exercises.

One strength of *The Maple V Handbook* is that examples are provided for each entry. Many of the examples are posed in a mathematical context similar to that found in an undergraduate textbook. This should be particularly beneficial to faculty and students who use Maple in their classes. On the other hand, some examples are so trivial that they add very little to the user's knowledge. For example, the only illustrations provided for standard mathematical functions (e.g., `arctan, cos`, and `BesselJ`) are exact and approximate evaluation and simple plots. These entries should mention, at a minimum, the domain, range, automatic simplifications and other technical information (e.g., branch cuts).

Also included in this handbook are a list of references (2 pages) and a convenient one-page pull-out Quick Reference. Noticeably absent from the book is any mention of the Maple Share Library (a public domain archive of user-contributed additions to and applications of Maple, available via the WWW at the URL: `ftp://ftp.maplesoft.com/pub/maple/share`); the Maple Users Group (an e-mail based mailing list, archived on the WWW at the URL: `ftp://daisy.uwaterloo.ca/pub/maple/MUG`); and *The Maple Technical Newsletter* (a refereed journal for Maple-related articles, editor: Tony Scott, `tscott@maths.ox.ac.uk`).

While this review has not included a careful proofreading of the entire text, a few typographical errors were found (e.g., $2^i - 4$ should be $2i - 4$ on p. 81, the running headings for Chapter 9 should be "The Linear Algebra Package: `linalg`", and the index entry for `evalhf` is misspelled on p. 717). Also, the two subsections in Chapter 13 (`plots` package) are out of place; these options are part of the built-in `plot` and `plot3d` commands described in Chapter 2.

While there is much useful information in *The Maple V Handbook*, it is my opinion that this book does not meet its stated objectives. While the ideal reference book does not yet exist, there are better handbooks than the one reviewed here. Another book, also by Abell and Braselton, that appears to be better organized is *Maple V by Example* [1]. Any reader who works through the examples in [1], which are different from the ones in *The Maple V Handbook*, would quickly become a skilled Maple user.

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47[11-02, 11Gxx]—*Arithmetic of algebraic curves*, by Serguei A. Stepanov , (translated from Russian by Irene Aleksanova), Monographs in Contemporary Mathematics, Consultants Bureau, New York, 1994, xiv+422 pp., 26 cm, $115.00

Based on a lecture series given at the Tata Institute of Fundamental Research, this book was intended to give a full picture of the state of the art in Diophantine equations. However, owing to space limitations, the author had to leave out analytic aspects connected with the circle method of Hardy and Littlewood. Also the logical aspects in connection with Hilbert's 10th problem had to be limited in size. Even within the remaining techniques from arithmetic algebraic geometry the author had to make choices. What remains, roughly speaking, is an extensive treatment of points on curves over finite fields, the study of integral points on curves over algebraic number fields and a discussion of Hilbert's 10th problem in a (too short) Appendix.

In the study of points on curves over finite fields the author has made an important contribution by giving an elementary proof of the Riemann hypothesis for the zeta-function of such a curve. This proof has later been streamlined by E. Bombieri who used the Riemann-Roch theorem rather than Stepanov's original estimates. This adapted proof, which cannot be found in many other books, is presented in Chapter 5. Before getting to that level, the reader in the preceding chapters was already led through several excursions into topics of number theory. For example in Chapters 1 and 2 we find a treatment of exponential sums and their estimates. In particular, we find a full discussion of Burgess's inequalities which, again, is not often found in other books. One of the interesting consequences of Burgess's estimate is that the least quadratic nonresidue modulo a prime p is bounded by $O_\varepsilon(p^{1/4\sqrt{e}+\varepsilon})$ for any $\varepsilon > 0$. The estimates of exponential sums are based on the estimates arising from the Riemann hypothesis for curves and techniques from Vinogradov.

The third chapter deals with rational points on algebraic curves of genus zero and one. The approach is very much in the spirit of Mordell's book on Diophantine equations and scratches only the surface of the subject. For example, in the case of genus-zero curves, the theorem that an odd-degree curve defined over the rationals has infinitely many rational points is not mentioned. As for genus-one curves, it is clear that one cannot give a concise overview of the theory of their rational points in a single chapter. The fourth chapter contains an introduction into the geometry of curves and the Riemann-Roch theorem. Unlike the previous chapter, this chapter is closely analogous to standard algebraic geometry texts on the subject.

The second main theme of the book is on integral points on plane algebraic curves. Suppose the curve C is given by the equation $f(x, y) = 0$ with $f \in \mathbf{Z}[x, y]$. We look for points $(x, y) \in \mathbf{Z}^2$ on this curve. A classical theorem of Siegel states that the number of such points is finite if genus(C) is positive or if C has at least three distinct points at infinity. This statement can be generalized to algebraic number fields and their integers and to so-called S-integers, where primes from a fixed finite set are allowed (Siegel-Mahler theorem) in the denominator. The proof of Siegel's theorem contains two ingredients which make the theorem ineffective, that is, it does not give a procedure to compute the finite set of solutions. The first is the rank of the rational points on the Jacobi variety of C, the second is the use of Siegel's method in Diophantine approximation. As suggested by Robinson and Roquette, it is possible to eliminate the ineffective part arising from the rank computation by use of methods from nonstandard arithmetic. It is this proof which is given in Chapter 7 of the present book. Unfortunately, nonstandard analysis/arithmetic is not in the reviewer's area of expertise, so I cannot make any comments here.

As a final remark I should say that the author has relegated much of his material to exercises, which are abundant and form a valuable part of the book. Some of them are quite hard. However, it is certainly worthwhile for a number theorist to browse through them. The author has collected therein many small interesting facts, and they pose interesting challenges for the reader or his or her students.

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