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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

5[65-02]—*Acta numerica 1995*, A. Iserles (Managing Editor), Cambridge University Press, Cambridge, 1995, 491 pp., $25\frac{1}{2}$ cm, $59.95

This is the 1995 issue of the annually published series Acta Numerica, which was started in 1992, but which does not appear to have been reviewed earlier in Math. Comp. The managing editor of the series is Professor A. Iserles of the University of Cambridge, England, who at his side has an editorial board of about 10 leading numerical analysts. The ambition, as expressed in the preface to the -92 volume, is to '...counteract the information explosion by presenting selected and important developments in numerical mathematics and scientific computation on an annual basis', and this is done by inviting authors on the basis of their contributions to the area and their excellence in presentation to write survey papers on relevant topics. In my opinion this is a very worthwhile purpose, and the present volume contains a convincing selection of such surveys. I could imagine that it may become increasingly difficult to find suitable topics and expositors as the years progress.

The present volume contains 7 articles and begins with a very well written presentation by P. T. Boggs and J. W. Tolle of *Sequential Quadratic Programming*, which is a class of iterative methods for solving nonlinearly constrained optimization problems employing quadratic subproblems. The authors first introduce the basic method, under the appropriate assumptions, and then discuss local and global convergence relating the algorithms to Newton's method. Several problems regarding implementation are also touched upon.

In *A Taste of Padé Approximation*, C. Brezinski provides an introduction to Padé approximation and related topics, with emphasis on questions relevant to numerical analysis and applications. Algebraic properties, including the relation with orthogonal polynomials and recursive computation, convergence theory, some generalizations, and applications, e.g., to A-acceptable approximations of ordinary differential equations are presented in separate sections.

Introduction to Adaptive Methods for Differential Equations, by K. Eriksson, D. Estep, P. Hansbo, and C. Johnson, is an enthusiastic account of Johnson's program for the design, analysis, and implementation of computational methods for differential equations, based on Galerkin finite element techniques and adaptivity. The framework is proposed to be accessible enough to be suitable for integration into basic calculus courses, thus fulfilling what the authors refer to as Leibniz's vision.

The paper *Exact and Approximate Controllability for Distributed Parameter Systems*, by R. Glowinski and J. L. Lions, is the second part of what amounts to a monograph on control of evolutionary partial differential equations, the first part of which appeared in Acta Numerica 1994. With its over 170 pages, or more than a third of the present volume, it is by far the longest paper and exceeds perhaps the ideal length of a survey paper for this series. After the groundwork laid in the -94 volume on control of linear diffusion equations, the authors continue in a systematic way to cover boundary control, control of Stokes systems and nonlinear diffusion equations, and control of wave equations and coupled systems. The different sections begin with theoretical considerations and continues with discretization with respect to both time and space, iterative techniques, and finally the results of a large number of numerical experiments.

Numerical Solution of Free Boundary Value Problems, by T. Y. Hou, is concerned with recent advances in developing efficient and stable numerical methods for problems with propagation of free surfaces, such as water waves, boundaries between immisible fluids, vortex sheets, Hele-Shaw cells, thin film growth, crystal growth and solidification. The paper first discusses locally well-posed problems using the boundary integral method, with application to water waves, and in a second part such methods are applied to ill-posed problems of fluid interfaces, mentioning in particular vortex sheets and associated singularity formation. In further sections the stabilizing effect of boundary tension is considered, as well as problems for which the boundary integral method is not suited.

Particle Methods for the Bolzmann Equation, by H. Neunzert and J. Struckmeier, discusses in a rather informal way numerical simulation of rarified gas flows by particle methods. After a section on collision integrals, particle methods are introduced first for the spatially homogeneous and then for the inhomogeneous Bolzmann equation. Practical aspects, relating to collision pair selection, stochastic methods, use of singular limits, domain decomposition, and use of parallelism are touched upon and some numerical results presented.

The New qd Algorithms, by Beresford N. Parlett, presents recent work, in which the author has been involved, concerning methods for computing eigenvalues and eigenvectors of tridiagonal matrices. The new algorithms discussed are based on factoring such tridiagonal matrices into bidiagonal ones and on Rutishauser's LR algorithm.

The numerical mathematics community will look forward to the 1996 volume of Acta Numerica with anticipation.

Vidar Thomée

6[65-01, 65M06, 65N06]—*Numerical solution of partial differential equations*, by K. W. Morton and D. F. Mayers, Cambridge University Press, Cambridge, 1994, 227 pp., $22\frac{1}{2}$ cm, hardcover, \$54.95, paperback, \$22.95

This is a book that grew out of undergraduate courses at Oxford. The emphasis is on traditional finite difference theory and the use of stability (and consequent emphasis on truncation errors). Stability is considered in L_∞ for parabolic and elliptic problems (using maximum-principles) and in L_2 (von Neumann-Fourier analysis) for hyperbolic and parabolic problems.

The presentation, naturally given in model situations for a course of this nature, is very clear and precise with mention (and most often some analysis) of key practical issues such as curved boundaries.

I believe that, in a first course of this nature, the Lax Equivalence Theorem should come with warning labels: "Do not thoughtlessly discard unstable methods."

(E.g., the Lax-Wendroff method is unstable in all L_p-spaces except $p = 2$.) Its main practical importance is, after all, that many methods which are unstable in $\underline{L_2}$ are "exponentially unstable" and useless.

I further believe that a course of this nature should devote some space to the construction of finite difference methods on irregular meshes, e.g., the control volume method. (Their analysis would be too far afield.) As it is, irregular meshes are presented only in a short section on the finite element method. However, the flashy cover picture is for a highly irregular mesh, as are all those nice pictures given in the introduction to grab the student's attention!

One can always argue with the material selected; I have taught a similar course at Cornell and then placed more emphasis on modern fast iterative methods (and, irregular meshes). By sticking to the straight and narrow in carefully selected material rather than painting with a broad brush, this book should succeed in imparting a critical attitude and feeling to the students for what should or should not be done in a practical solution.

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7[65-01]—*Computer Numerik 1, 2*, by Christoph Überhuber, Springer, Berlin, 1995, (Part 1) xvi + 511 pp., (Part 2) 515 pp., $23\frac{1}{2}$ cm, softcover, DM 78.00

Although, formally, *Computer Numerik* by Christoph Überhuber comes in two parts, both volumes should be regarded as a whole. *Computer Numerik* thus is a comprehensive, 1000 page textbook on numerical computing for practitioners. It bridges the gap between mathematics dominated treatises on numerical analysis on the one hand and recipe-type collections of numerical programs on the other hand.

The first 8 (out of 17) chapters of *Computer Numerik* discuss general aspects of numerical computing like the modelling process, basic numeric concepts, sources and types of errors, modern computing platforms (hardware and languages) and basic issues on commercial as well as public domain software for numerical computing. Modelling related issues and error identification are given particular emphasis and the author develops a fairly elaborate, detailed, sometimes novel (not always standard) terminology and concept formation in this context.

Chapters 9 to 17 deal with different problem classes for which existent numerical software has reached a sufficient state of maturity: Approximation, interpolation, Fourier-transforms, evaluating integrals, linear and nonlinear systems of equations, the eigenproblem, sparse matrices and random number generation. Each of these topics is approached in a way that particularly fits the needs of the practitioner: Mathematical *concepts* are explained in detail while *proofs* or more involved *numerical details* in the algorithms are left out. Limitations and finite precision issues in each problem class are identified so that the reader gets a clear impression of what can realistically be achieved. For each problem class the book points to the relevant available software in commercial numerical libraries like NAG and IMSL and to corresponding state-of-the-art public domain software packages like LAPACK, QUADPACK or ITPACK. All subjects are illustrated by various (more than 500 in total!) impressive and non-trivial example applications. However, I would have liked to see one more chapter on validating numerical techniques (which take round-off into acount) and corresponding software instead of the one and only very short remark on that subject in Chapter 4.

If numerical analysts have been complaining that their admitted high quality public domain software packages do not yet have the deserved impact on the 'end-user', this book will certainly contribute to change the situation. But it is more than just a guide to numerical software: It is a fundamental work on numerical computation which makes many major achievements in numerical analysis available to the practitioner.

An English translation of the book is under preparation.

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8[68Q05, 68–06]—*Abstract machine models for highly parallel comptuers*, John R. Davy and Peter M. Dew (Editors), Oxford University Press, New York, 1995, xii + 337 pp., 24 cm, $80.00

For sequential computers the von Neumann model has served the needs of practitioners and theoreticians for a long time. In its simplest form a von Neumann computer executes instructions one at a time in a fetch-execute cycle. First an operation code and operand data are fetched from memory, then the corresponding instruction is executed and the result is sent back to memory for storage. Despite the many elaborations of this simple idea in actual computer hardware, reality has been represented closely enough by this model for some purposes. On the practical side, software and hardware designers have been able to advance their own craft without need for a detailed understanding of the opposite craft. And theoreticians concerned with such issues as complexity analysis of algorithms have also been able to make effective use of the model.

The model breaks down when multiple processors and memories are linked together in a parallel computer system. A variety of replacement models have emerged but often these are positioned too closely to either the software or the hardware end of the spectrum, with the result that the main advantages of the von Neumann model (simplicity and generality) are lost. This book contains 18 papers presented at a Workshop on Abstract Machine Models for Highly Parallel Computers which was held at Leeds University in April, 1993. The purpose of this workshop, the second on the subject, was to consider whether a model can be devised to bridge the hardware-software gap somewhere near its center. Interestingly, relatively concrete proposals have been made that appear to do this quite well. This book provides a valuable cross section of work in this important area of computer science.

Daniel W. Lozier

9[86-06]—*Mathematics, climate and environment*, J.-I. Diaz and J.-L. Lions (Editors), Research Notes in Applied Mathematics, Vol. 27, Masson, Paris, 1993, 315 pp., 24 cm, softcover, F 320

There are many books on water waves that are a successful blend of high quality mathematics and physics. With regards to mathematics, climate, and environment, however, books written in the same spirit are difficult to find. Among the climate-related books, M. Ghil and S. Childress' "Topics in Geophysical Fluid Dynamics:

Atmospheric Dynamics, Dynamo Theory, and Climate Dynamics", published by Springer-Verlag in 1987, and "Physics of Climate" by J. P. Peixoto and A. H. Oort, published in 1992 by the American Institute of Physics, come to mind as examples of books that present the scientific issues in appropriate detail and that emphasize the most important character of climate/meteorology and environmental dynamics from a physical and mathematical point of view: they are the result of the complex interaction of a multitude of physical phenomena. Both of these books are required reading for mathematicians working on climate problems. However, neither of these books' authors intended to address the issues from a mathematical standpoint.

There is great demand on the part of applied mathematicians for books that take a comprehensive view of the subject of mathematics and the environment. A publication in this category could have been "Mathematics Climate and Environment", a book which introduces a few of the main climate and environmental issues and many of the mathematical techniques used to study these issues to a mathematically-inclined audience. However, this book cannot be included in this category and, in my opinion, this situation could have been avoided by more diligence with regards to presentation on the part of the Editors, and by more attention to content by a couple of the contributors.

The book is a meeting proceedings. It comprises eleven lectures and twelve short talks which were presented at the "Summer Course of the Universidad Complutense: Mathematics, Climate, and Environment", held in El Escorial, Spain, in August of 1991. There were presentations on pollution control, global and regional climate, meteorology, traffic flow, stationary solutions to Navier-Stokes equations, and even the disaster at Chernobyl. With regards to mathematics, topics were drawn from large-scale computation; empirical analysis and statistics; optimal control, sensitivity analysis, inverse problems, and data assimilation; inertial manifolds, and global attractors; analysis, especially solutions smoothness, uniqueness, stability, and well-posedness of equations. Some of the lectures and talks were delivered by leading authorities in their respective subspecialty, for example, J. L. Lions covered approximate and optimal controllability of nonlinear systems; R. Temam had a nice introduction on inertial and slow manifolds, G. North, J. Diaz, I. Stakgold presented energy-balance models, Marchuk and Le Dimet discussed sensitivity analysis and data assimilation, A. Ruiz de Elvira covered statistical techniques in the analysis of signals from nonlinear systems. M. Ghil apparently (the manuscript for this lecture is inexplicably limited to the abstract) spoke on dynamical systems techniques in the analysis of field data.

The shortcomings of the book are mostly editorial in nature, however, in a couple of the lectures, there are problems with content as well. The Editors should have availed themselves of a copy editor in order to substantially improve the translations, to avoid the excessive number of typographical errors, and to typeset the manuscript using a single text processing software package. Many of the books' figures are poorly reproduced and, in some instances, were not properly commented upon in the accompanying text. One lecture does not contain a reference list, others have irritating typographical errors in their bibliographies.

With regards to content, the most salient problem is that a couple of the authors who alluded to modeling physical phenomena did so in a very cursory or simplistic manner. While it is certainly true that the main emphasis of these lectures is mathematical rather than physical, some would argue that disregard for the issues of modeling severely undermines some of the reasons for studying these models from a mathematical standpoint in the first place.

In summary, "Mathematics Climate and Environment" is an attempt, as the Editors say in their preface, to cover some of the basic issues of climate and the environment for the mathematically-inclined scientist. For the non-specialist, some of the papers are good starting points on several of the topics. Many will appreciate the expository style of most of the lectures, avoiding excessive technicalities in the presentation. However, rapid progress in some areas of climate and the environment (of which some of the authors in the book can take responsibility) and a better definition of the role mathematics research plays in contributing to furthering our understanding of climate and the environment, makes some of the material that appears in the book somewhat dated. Alas, when compared to current research, the book is a testimonial of what makes this line of research so exciting to those of us who work in the field.

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10[65F10, 65H10, 65H20]—*Iterative methods for linear and nonlinear equations*, by C. T. Kelley, Frontiers in Applied Mathematics, Vol. 16, SIAM, Philadelphia, PA, 1995, xiv + 165 pp., $25\frac{1}{2}$ cm, softcover, $32.50

This volume gives an introduction to the topic of iterative algorithms for systems of algebraic equations. It gives an overview of a few methods for linear systems followed by a somewhat deeper summary of the main approaches for solving nonlinear systems.

The contents are as follows. The first three chapters concern linear equations. Chapter 1 introduces some basic concepts such as splitting operators. Chapter 2 gives an introduction to Krylov subspace methods and an overview of the conjugate gradient method, applicable to symmetric positive systems or the normal equations associated with other systems. Chapter 3 discusses methods for nonsymmetric systems with emphasis on GMRES. For both classes of methods, the presentation begins with theoretical properties (minimizing functionals over subspaces), and this is followed by discussions of implementation and some examples of numerical performance on model problems. The rest of the book (five chapters) concerns nonlinear equations. Chapter 4 presents introductory material such as fixed point methods and rates of convergence (linear, superlinear, etc.). Chapter 5 concerns Newton's method and simple variants such as chord methods which reuse the Jacobian matrix. Chapter 6 concerns inexact Newton methods in which the Jacobian systems are not solved exactly, but instead inner iteration is used to compute approximate solutions. Chapter 7 discusses Broyden's method as an example of quasi-Newton methods, which construct approximation to the Jacobian matrix. There is convergence analysis for all of these methods. Chapter 8 presents criteria on step-lengths to ensure global convergence.

I believe this book will be valuable as a reference and of some use as a graduate text for a "topics" course in iterative methods. It is very up-to-date and provides pointers to MATLAB software available through the World Wide Web or anonymous ftp. The exposition benefits from the author's decision to sacrifice some generality when simpler arguments are available. Pointers to an extensive bibliography provide additional details. Many "difficult" topics such as stopping criteria and implementation issues are discussed, and the experimental comparisons running through the book are useful. As a text it will probably need to be supplemented, especially for linear systems. (For example, there is nothing about symmetric indefinite linear systems.) I downloaded and ran one example of the software (GMRES) and found it easy to use as well as to modify.

The one flaw in the book concerns its editing. I noticed enough typographical and grammatical errors to prompt me to start keeping a running count; I observed ten such errors in one thirty-three page interval. Otherwise, the book is well-written and I believe well-suited as an introduction to this material.

Howard Elman

11[11R11, 11R29, 11Y40]—*Quadratics*, by Richard A Mollin, The CRC Press Series on Discrete Mathematics and Its Applications, CRC Press, Boca Raton, FL, 1966, xx + 387 pp., 26 cm, $74.95

The title refers to any and all aspects of quadratic fields. The author has written a large number of papers (many in collaboration with H. C. Williams) and this book serves very well as a platform for related topics. The most obvious purpose fulfilled by this book (and by none other at present) is the large bibliography with references of hundreds of papers, incidental or central to quadratic fields.

The book tends to accept the classical theory as a necessary evil in order to present the exotica, and indeed this book is not meant for the unacquainted to learn about quadratic fields. Nevertheless, a reader with only a casual acquaintance with quadratic fields will find very many rewarding byways. For this reader, the book is best appreciated at first contact by browsing all the way from beginning to end, and then going back. The review is written in this spirit, with a somewhat arbitrary choice of topics.

[The reviewer's reference to "exotica" among the contents is not necessarily negative. At one time, e.g., nonunique factorization was more or less in this category, yet it became the primary challenge of "main line" Number Theory.]

The author favors the ideal-theoretic approach very strongly but the reader should also understand that there is a case for quadratic forms which is characteristically *quadratic* rather than a special case of fields of arbitrary degree:

The composition of quadratic forms. *The theory of* (*binary*) *Quadratic Forms is nothing but the study of the* (*Gauss*) *composition identity*

$$(a_1x_1^2 + bx_1y_1 + a_2cy_1^2)(a_2x_2^2 + bx_2y_2 + a_1cy_2^2) = (a_1a_2x_3^2 + bx_3y_3 + cy_3^2),$$

with variables satisfying a bilinear relation over $\mathbb{Z}[a_1, a_2, b, c]$, *namely*

$$x_3 = x_1x_2 - cy_1y_2, \qquad y_3 = a_1x_1y_2 + a_2x_2y_1 + by_1y_2.$$

This identity involves three quadratic forms of discriminant $d = b^2 - 4a_1a_2c$, and the relation to algebraic number theory comes from the norm "N" in

$$ax^2 + bxy + cy^2 = N(ax + (b + \sqrt{d})y/2)/a, \ (d = b^2 - 4ac).$$

So the forms are associated with the modules like $\{ax + (b + \sqrt{d})y/2 : x, y \in \mathbf{Z}\}$. Ideal theory enters in a natural way and is a tool for easy generalization to fields of arbitrary degree, once the bilinear concept is utilized.

One step beyond the composition identity, the object is to represent a number by a quadratic form, and the identity shows that if the prime factors are represented by forms, the factors can be put together to represent the number by multiplying forms. Forms under unimodular equivalence represent the same numbers, so this equivalence class concept is natural. Under the equivalence class concept, any two forms of the same discriminant have equivalent forms satisfying such an identity (possibly with minor restrictions in case $\gcd(a_1a_2, d) \neq 1$).

For Gauss this was a system which circumvents nonunique factorization. For instance, in the classic case $\mathbf{Q}(\sqrt{-5})$ there are two factorizations of 9 manifesting nonuniqueness,

$$3 \cdot 3 = (2 + \sqrt{-5}) \cdot (2 - \sqrt{-5}).$$

These factorizations correspond to two inequivalent forms with $d = -20$,

$$\Phi_1(x, y) = (x^2 + 5y^2), \qquad \Phi_2(x, y) = (2x^2 + 2xy + 3y^2),$$

(inequivalent since $\Phi_1(1, 0) = 1$, but $\Phi_2(x, y) \neq 1$ for all $x, y \in \mathbf{Z}$.) The equivalence classes form a group of two elements: $\Phi_2^2 = \Phi_1$, viz.,

$$\Phi_2(x, y) \cdot \Phi_2(x - y, y) = \Phi_1(2x^2 + 2y^2, y^2).$$

From here, the theory goes to the multiplication of modules and finally to ideal class structure. This is the classical motivation for "quadratics".

The author, reversing history, develops ideals and makes quadratic forms just an appendix item. In terms of parameters and coordinates, the modules are more useful as a concrete practical model for the forms.

Of course, this does not mean the book is insensitive to history. To the contrary, there is a parallel commentary in footnotes and text which is very engaging and is worth reading, even for experts. In some places, the footnotes on main line history are in contention with the text. Anyway, the author is in the game for exotica, much of which is in topics featuring his own research and that of his students.

In each chapter, after he disposes of theory (sometimes very quickly) he gets down to business. The exercises are very helpful as they serve as an excellent (and necessary) method of understanding the proofs and his favorite techniques.

In Chapter I (Algebraic Number Theory), "powerful" numbers seem to come at us out of nowhere. These are numbers of the form $n = x^2y^3$ (alternatively $p \mid n \Rightarrow p^2 \mid n$). The first interesting theorem on them is that for every $(0 \neq)m \in \mathbf{Z}$, the diophantine equation $x^2y^3 - u^2v^3 = m$ is solvable infinitely often. If $m(= 2t+1)$ is odd, then trivially $m = (t+1)^2 - t^2$, but the *infinitude* is not trivial. The author's clever trick is to devise $Ar^2 - Bs^2 = m$ and then to solve $T^2 - rsU^2 = \pm 1$. Then an infinitude of exponents k are found (in a Fermat-type congruence class) such that

$$(A\sqrt{r} + B\sqrt{s})(T + U\sqrt{rs})^k = gr\sqrt{r} + hs\sqrt{s}.$$

The norm operation completes the proof. This method leads to nine special cases with A, B, r, s, T, U parametrized by m and k. The author then proceeds to a plethora of theorems, lemmas, and conjectures. The effect seems a "bit much", but it is somewhat justified by the apparent connection of the above equation with the conjecture of Ankeny, Artin, Chowla and of Mordell that the coefficient of $\sqrt{p}$ in the fundamental unit for $\mathbf{Q}(\sqrt{p})$ is not divisible by p (prime). The conjectures have a role in class field theory (which is usually considered to be main line rather than exotic).

Chapter II (on Continued Fractions) is set up in great detail as one of the author's main tools; the complex case is included. Cases of short period for $\mathbf{Q}(\sqrt{D})$, $D = s^2 + r$, $r \mid 4s$, (called "ERD" for "Extended Richaud-Degert") are enumerated in detail in Chapter III (Diophantine Equations and Class Number), and are tools which do not have long to wait for use as illustrations with which we *can actually calculate*. The exercises also cover other more classical exercises such as Lucas-Lehmer Theory and the equation $x^2 - D = p^n$ for fixed $D < 0$ and p prime. (The continued fractions of the basis of a real quadratic field were given the attractive name, "the Infrastructure" by Shanks.)

Here we come to a serious bit of exotica, due to Weinberger and Yamamoto. Let a be odd and let $D = a^{2n} + 4$ be primitive (i.e., D/g^2 is not a discriminant for any $g > 1$). Then $\mathbf{Q}(\sqrt{D})$ has class number divisible by n. (Indeed, the divisors of a have order divisible by n.) There are also infinitely many such D so *there is an* (*explicit*) *infinitude of quadratic fields of class number divisible by any given* n. This is an illustration of the frequent trick of using the ERD type of radicand $D = s^2 + 4$.

Chapter IV (Prime-Producing Polynomials) and Chapter V (Class Numbers: Criteria and Bounds) produce the main chain of connected results. The Euler-Rabinowitsch polynomials are

$$f(x) = x^2 + x + m, \qquad m \in \{1, 2, 3, 5, 11, 17, 41\},$$

with the property that $f(x)$ is prime for $x = 0, \ldots, m-2$ (vacuously true for $m = 1$). The inference that $\mathbf{Q}(\sqrt{1-4m})$ has class number one is an exciting result, particularly since the connection with class number works both ways. Ultimately, similar (prime-producing) polynomials come at us en masse (again diluting the excitement of the original polynomials), but leading to criteria of class number one and bounds on the class number. These bounds acquire a special urgency from Gauss's conjecture that for real quadratic fields, the class number may equal one for infinitely many (prime) discriminants.

The most prolific bounds come from the GRH (Generalized Riemann Hypothesis) on the zeros of L-functions as well as ζ-functions. Here, not unexpectedly by now, a lemma (of Tatuzawa) comes out of nowhere, giving

$$L(1, \chi) > .655\varepsilon D^{-\varepsilon},$$

for $.5 > \varepsilon > 0$ and $D > \max\{e^{1/\varepsilon}, e^{11.2}\}$, with D the fundamental discriminant for the L-function. This in turn creates an easy lower bound on the class number, which has $L(1, \chi)$ as a factor in the famous Dirichlet formula.

It would be enlightening to most readers to show even by sketchy heuristics how this inequality is related to the GRH, but the author is too intent on applying this bound, and apply it he does. We have a set of interlocking conjectures of Yokoi, Mollin, and Williams which succeed in establishing that if we restrict our discussion to the (more tractable) ERD-cases, then for discriminant > 1757 the class number exceeds one (unless one exceptional discriminant comes from the failure of the GRH).

Chapter VI (Ambiguous Ideals) deals with ambiguous (self-conjugate) ideals and ideal classes. The problem is usually restricted by starting from equivalent quadratic forms. The author approaches it in full generality, using his method of palindromic continued fractions. There are applications to statements of when the class group is a 2-group. Chapter VII (Infrastructure) discusses a concept of Shanks, namely the "quadratic residue cover" for a function, i.e., a finite set of primes such that every value of the function is divisible by or is a residue of one of the primes. If the function is a sequence of discriminants, there is ready-made information on nonprincipal ideals, which relates to the class number.

Chapter VIII (Algorithms) is a nonidiosyncratic discussion of factorization and primality. The final section of the text, however, on Computation, is a thoughtful commentary, (basically accepting computers as a fact of life). It also almost reads like a personal testament of the author, but for that matter so does the whole book.

This book is followed by an appendix of 85 pages of numerous tables for units and class numbers, etc. There are also commendably dozens of tables in the text illustrating many of the theorems. These show the author's intense desire to inspire experimentation and reader participation.

Yet mathematics books also have browsers, who want to open a book and come up with a result (maybe also with a proof), just the way a brewery has visitors who do not want to buy the brewery, but just want to enjoy a free beer. The author does not make it easy for such casual readers. Most results have some nonstandard symbol, abbreviation, or neologism which requires further cross-references by the reader, perhaps causing the uncommitted reader's curiosity to wane.

The classical style once required that the more important the theorem the fewer the symbols and plainer the prose. The reviewer wishes the author were more so inclined, but the book is still well worth the effort (and the extra effort).

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12[11A41, 11-04, 11N36, 11Y60, 68M15]—*Enumeration to* 10^{14} *of the twin primes and Brun's constant*, by Thomas R. Nicely, Virginia Journal of Science **46** (1995), 195–204

The set $S = \{(3,5),(5,7),(11,13),\dots\}$ of twin prime pairs has been studied by Brun (1919) and more recent authors. It has never been proved that S is infinite, although the appropriate Hardy-Littlewood conjecture and numerical evidence strongly suggest that it is. Brun showed that the sum B of reciprocals of twin primes converges (unlike the sum of reciprocals of all primes). However, the sum defining B converges very slowly and irregularly.

Nicely's paper gives counts $\pi_2(x)$ of the number of twin prime pairs $(q, q+2)$ such that $q \le x$, and the corresponding sum of reciprocals $B(x)$, for various $x \le 10^{14}$. The most extensive previously published computation, by the reviewer (1976), went only to $x = 8 \times 10^{10}$.

Many of Nicely's values of $\pi_2(x)$, including those for $x = 10^{13}(10^{13})10^{14}$, have been confirmed in an independent computation performed by J. Kutrib and J. Richstein (personal communication from J. Richstein, September 21, 1995). Similarly for $B(x)$ (to at least 16 decimal places). We record Nicely's values

$$\pi_2(10^{14}) = 135780321665$$

and

$$B(10^{14}) = 1.82024496813027052889471783861953382834649$$

(believed to be correct to all places given). By extrapolation using the Hardy-Littlewood conjecture, Nicely estimates

$$B = 1.9021605778 \pm 2.1 \times 10^{-9},$$

where the error estimate is not a rigorous bound but corresponds to one standard deviation using a statistical model based on some plausible assumptions. This is consistent with the previous estimate, by the reviewer, of $B = 1.9021604 \pm 5 \times 10^{-7}$ (using a more conservative methodology for the error estimate). A better estimate, based on a subsequent computation to $x = 2.5 \times 10^{14}$, is

$$B = 1.9021605803 \pm 1.3 \times 10^{-9}$$

(personal communication from T. Nicely, April 4, 1996).

Nicely's computations were performed on Intel 80486 and Pentium computers; the sieving speed on a Pentium was about 10^{11} integers per day. In such an extensive computation there is a significant probability of error. The author deserves full credit for his careful error checking which uncovered several potential sources of error, including logical flaws, compiler/library bugs, disk and memory problems, and, last but not least, a design flaw in the floating-point unit on certain Pentium chips. The latter received widespread publicity. On December 20, 1994, Intel offered to replace faulty Pentium processors free of charge.

Nicely's paper is of general interest as a warning of how careful one needs to be in a very extensive computation. Anyone who is interested in large-scale computations should be able to learn something of value from the paper. Computer hardware designers and compiler writers might also learn some useful lessons.

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