MATHEMATICS OF COMPUTATION
Volume 66, Number 219, July 1997, Pages 1367–1374
S 0025-5718(97)00877-6

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

13[65-06, 11-06, 11Yxx]—*Mathematics of Computation 1943–1993: A half-century of computational mathematics*, Walter Gautschi (Editor), Proc. Sympos. Appl. Math., Vol. 48, American Mathematical Society, Providence, RI, 1994, xx + 643 pp., 26 cm, $90.00

Already in my student days, *Mathematics of Computation* was one of my favorite journals. Every issue would start with a thick layer of "other people's mathematics", whose sole function, as I imagined, was to keep its real contents hidden from general view. After many pages of finite element methods and quadrature formulas one would, at an always sharply defined point, enter the magical world of primality tests, amicable numbers, and sieving devices—true and tangible mathematics that held an immense fascination for me. No other journal published a higher percentage of papers that I found so immediately appealing. In fact, no other journal published any papers whatsoever in computational number theory, and were it not for *Mathematics of Computation*, I might have thought there was nothing respectable about the entire discipline at all.

Much has happened in computational number theory since I was a student. Theoretical computer scientists discovered it as a playground, invented new methods and reinvented old ones, established the habit of not only *running* algorithms but also *analyzing* them, and have now largely left the field again. Cryptologists found that problems in computational number theory display the right combination of feasibility and intractability that is required for the construction of cryptographic schemes that are both practical and secure—that is, until someone discovers that there is nothing intractable about these problems after all. And, thirdly, researchers in other branches of number theory started using computers as a research tool and developed algorithms in areas where none or few had been before.

Many traditional mathematical journals, both the arithmetically oriented ones and those of a more general signature, responded to the increased fashionability of computational number theory with a notable change in attitude. In addition, computer scientists and cryptologists founded their own journals, and in pure mathematics *Experimental Mathematics* made its appearance. As a result, *Mathematics of Computation* lost its monopoly, but it has done much better than merely surviving. It is universally regarded as the leading journal in computational number theory, representing the full range of current research in the field, and remaining essentially the only one for the purely "numerological" aspects.

In 1993, the 50th birthday of *Mathematics of Computation* was commemorated with an International Conference held in Vancouver. It consisted of a Symposium on Numerical Analysis and a Minisymposium on Computational Number Theory. The latter was dedicated to the memory of D. H. Lehmer, of all of the founders of the journal the one to be held responsible for its reputation in number theory. The volume under review contains, in Part I, the proceedings of the Symposium, and in Part II those of the Minisymposium.

It is only Part II, which occupies almost a third of the book, that concerns us in this review. It comprises four invited papers and thirteen contributed ones. The latter have no more than six pages each; five of them are in final form, and of seven will a final version appear elsewhere, the status of the thirteenth, which deals with the philosophy of mathematics, being characteristically unclear.

The longest paper in the volume—51 pages, including 6 pages of references—is the "historical essay" *Factoring integers before computers*, by H. C. Williams and J. O. Shallit. Number theorists with an interest in the history of their subject will love perusing this paper; and it must be considered required reading for scholars whose occupation with factoring integers is inspired by its relevance in cryptology. The security of many modern cryptographic schemes depends crucially upon the supposed intrinsic intractibility of certain problems in computational number theory, such as the problem of factoring large integers. It is all too often forgotten that the *only* evidence for the correctness of this supposition is of a historical nature. Whoever wishes to form an independent opinion of the strength of this evidence must study the history of the subject, and the paper of Williams and Shallit is the best place to start.

There is also an invited paper on what, in 1993, promised to be the near *future* of factoring integers. Carl Pomerance speculates that the "number field sieve" will emerge as the method of choice for factoring the hardest numbers, an expectation that has been borne out by the subsequent developments. His beautifully written paper forms an excellent introduction to modern factoring techniques, with special emphasis on the number field sieve. Andrew M. Odlyzko contributed a concise and lucid survey of analytic computations in number theory, with copious references. A fourth invited paper, by Ingrid Biehl and Johannes Buchmann, deals with algorithms in quadratic number fields, addressing a more specialized audience than the other three papers.

No reader whose favorite journal is *Mathematics of Computation* will want to miss this book, which provides them with as much fun as the journal itself does. Even the bivariate splines and Galerkin methods are not absent, to keep up the idea that it would be sinful to devote an entire volume to the pursuit of "big game in the theory of numbers".

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14[65-01]—*Afternotes on numerical analysis*, by G. W. Stewart, SIAM, Philadelphia, PA, 1996, x + 200 pp., $23\frac{1}{2}$ cm, $29.50

Here's numerical analysis with a lean and lively spirit. G. W. "Pete" Stewart has compiled for us a set of notes for an introductory course in numerical analysis. The terminology "Afternotes" is indicative of his practice of writing down his recollections of the lecture just given, while they were fresh in his mind, thus putting his own spin on the material. As befitting of a fledgling audience, this is not a traditional theorem-proof presentation. Rather, the intent is to get to the heart of the matter; the rigor and detail are out there if you want to look for them.

Given this framework, "Afternotes" is not intended to fulfill all of the needs of a full-fledged textbook. Exercises are few in number and not intended to supplant routine problems or computer projects. Fully two-thirds of the lectures are devoted to matters dealing with linear and nonlinear equations and floating-point arithmetic. Therein lies the strength of the book; Stewart knows just how to illuminate important computational issues, such as effects of conditioning. On the other hand, the remaining third of the lectures deal with standard fare, such as interpolation, numerical integration, and numerical differentiation, and here Stewart manages to hit the high spots with just the right effort for a course at this level.

Notably absent, then, is a section on initial-value problems for ordinary differential equations. One might argue that the course is already packed; nevertheless, inclusion in the notes of a section on ODE's to be used at the discretion of the instructor might be useful.

As befits an expert in the field, Stewart not only covers standard fare skillfully, but it is his introduction to, and treatment of, topics not always encountered at this level, e.g. perturbation theory and backward error analysis, which gives this work much of its value. Other examples of material presented here and not readily found in most texts include a linear-fractional method and a hybrid scheme for solving $f(x) = 0$, and an indication of the role of row vs. column orientation for algorithms for solving linear systems.

As in many texts, a goal here is to point out the nuances and possible pitfalls in numerical computation. This Stewart accomplishes, while at the same time injecting into the presentation an appropriate dose of humor. Notable examples include a spirited conversation between scientist Dr. XYZ and you, the reader, as the numerical analyst, discussing the backward error analysis of summation, and an admonition to "economists and astrologers" on the dangers of unsafe extrapolation (you'll have to read this one for yourselves).

To conclude, we have already seen some reasons (notably, lack of exercises), which prevent this set of notes from being the sole text for a course. We might also ask from the author some guidance concerning his expectation of the reader's knowledge of programming languages, since code segments in differing languages are interspersed throughout the text. But, in the final analysis, what we do have here is an excellent set of notes which might be used to reinforce materials from other sources, or, depending upon the audience, to use as primary lecture material, with a traditional text kept in the bullpen for more detail. Either way, read and enjoy!

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15[76-02, 65M60]—*Navier-Stokes equations and nonlinear functional analysis*, by Roger Temam, CBMS-NSF Regional Conference Series in Applied Mathematics, Vol. 66, second edition, SIAM, Philadelphia, PA, 1995, xiv + 141 pp., 25 cm, softcover, $26.50

This short monograph is volume 66 of the CBMS-NSF regional conference series in applied mathematics. It is the revised second edition of a text that was published in 1983 as volume 41 of the same series. The author states in the introduction that "the mathematical study of the Navier-Stokes equations is difficult and requires the full force of modern functional analysis. Even now, despite all the important work done on these equations, our understanding of them remains fundamentally incomplete". His aim while writing these notes was to "arrive as rapidly and simply as possible at some central problems in the Navier-Stokes equations", hoping that they would stimulate interest in these equations.

Most of the material presented in the first edition of the book is still relevant and has not been changed, except for the correction of a few (but not all) misprints. The revision consists in the addition of an appendix devoted to inertial manifolds and an update of the rather extensive bibliography (almost 200 items).

The book is divided into three parts, each addressing a group of topics of central importance. Part I addresses questions concerning existence, uniqueness and regularity. Temam begins by recalling the initial-boundary value problems associated with the Navier-Stokes equations (NSE). Then he presents a functional analytic setting for the equations, emphasizing the case of flow in two and three space dimensions with space periodic boundary conditions. This leads to many technical simplifications, while retaining the main mathematical difficulties (except, of course, those related to boundary layers). The modifications needed for physically relevant boundary conditions are given without proofs. This is followed by a presentation of the classical existence and uniqueness results based on the standard Galerkin approximation procedure. The classical *a priori* estimates are derived in detail and the compactness argument leading to convergence of the approximations is outlined. He then proceeds to discuss more advanced topics such as the fractional dimension of the set of singularities of a solution, analyticity in time, compatibility conditions at $t = 0$ (for non-periodic boundary conditions), and the Lagrangian representation of the flow.

Part II deals with the long-time behavior of solutions. Three topics are developed here: Temam first presents a theorem on the number of stationary solutions based on an infinite-dimensional version of Sard's theorem. Then he proves what he calls the "squeezing property": the flow is essentially characterized by a finite number of parameters. Finally he proves that functional invariant sets (e.g. attractors) have finite Hausdorff dimension.

Part III addresses questions related to numerical approximation. Temam presents a completely discrete scheme, that combines an alternating direction method in time with a finite element method in space. Then he gives a detailed convergence proof based on a new (as of 1983) compactness theorem. He also shows that the long-time behavior of eigenfunction Galerkin approximations is completely determined by a fixed finite number of its eigenmodes, a result that adds to the evidence that the long-time dynamics of the NSE is finite dimensional.

Finally, a new appendix is devoted to inertial manifolds. Roughly speaking, an inertial manifold of an evolution system is a smooth finite-dimensional manifold that attracts every solution exponentially fast. Thus, if an inertial manifold exists, then after an initial transient the dynamics of the system is essentially determined by a finite-dimensional system of ODE's, the inertial system. Temam first outlines the basic existence result for inertial manifolds, emphasizing the so-called spectral gap condition: there must be sufficiently large gaps in the spectrum of an associated linear operator. Noting that the gap condition fails for the NSE, Temam then presents a recent attempt to circumvent the difficulty, due to Kwak, that is based on embedding the 2D space periodic NSE into a larger system of reaction-diffusion type, for which the gap condition is less severe. However, after the publication of this book it was found that this argument is incomplete, and it is still unknown whether the NSE possess an inertial manifold or not. The gap in the proof is related to the fact that the linear part of the new system is no longer self-adjoint, as will be explained in the forthcoming second printing of the text.

I think that Temam has achieved his goal: using these notes a reader with a good background in functional analysis will go quickly to some advanced topics in the Navier-Stokes equations. With its ample bibliographical comments the text is also a nice survey of interesting questions, techniques, and results.

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16[76-02, 76D05, 76Mxx, 65Mxx]—*Numerical solution of the incompressible Navier-Stokes equations*, by L. Quartapelle, International Series of Numerical Mathematics, Vol. 113, Birkhäuser, Basel, 1993, xii + 291 pp., 24 cm, $100.00

The professed aim of this book is to "give a unitary (unified?) view of the methods which reduce the equations for viscous incompressible flows to a system of second-order equations of parabolic and elliptic type". Such methods are popular especially with engineers and have the advantage of avoiding the approximation of the incompressibility condition and the numerical stability issues associated with such approximations. These "nonprimitive variable" methods on the other hand suffer from the difficulty of providing meaningful boundary conditions for the new variables, such as the vorticity, that are introduced. Much of the original work contained in this book consists in deriving appropriate conditions which turn out to be of integral type. Additionally, the equivalence of these various formulations with the original primitive variable equations is demonstrated.

Chapter 1 is introductory in nature and contains a short but interesting discussion of the issue of compatibility of initial data. Chapter 2 is concerned exclusively with two-dimensional flows. The classical vorticity-stream function formulation is derived. The lack of boundary conditions for the vorticity ζ motivates the elimination of ζ and the formulation of a fourth-order (biharmonic) equation for the stream function ψ. Alternatively, integral vorticity conditions are derived which allow the uncoupling of ζ and ψ. The integral nature of these conditions is compatible with the fact that ζ is less regular than the velocity $\mathbf{u}$. Implementations of these integral conditions by means of finite difference, finite element and spectral methods are also given.

In chapter 3, the discussion is extended to three-dimensional flows. The technical issues are vastly different since both the vorticity and stream function are now vector variables. Indeed, there are six unknowns now instead of four in the original formulation.

Chapter 4 is devoted to vorticity-velocity formulations in both 2 and 3 dimensions. Even though there are several advantages associated with them, such "hybrid" formulations have remained relatively unexplored.

In chapter 5 a method for obtaining a system in the primitive variables $\mathbf{u}$ and p is presented. One starts from a semidiscretization in time of the Navier-Stokes equations by means of a time stepping method; then the continuity equation is eliminated to obtain a Poisson equation for the pressure. An integral condition for the pressure is then derived supplementing the system. Chapter 7 is devoted to a discussion of the fractional-step projection method for the primitive-variable Navier-Stokes equations. Chapter 8 is concerned with the incompressible Euler equations the emphasis being placed on discretizations by Taylor-Galerkin methods. Finally, a set of appendices provide expressions for vector differential operators in orthogonal curvilinear coordinates and other useful vector identities.

The book succeeds in achieving its intended goal. It gathers a wealth of useful information some of which is new while the rest is scattered in the literature. Although the mathematical background required is such that the book is accessible to students and beginning researchers, a significant amount of the material included can be only appreciated by the more experienced practitioner. There are however a great number of typos and minor grammatical offences which fortunately do not manage to destroy the otherwise flowing narrative.

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17[65-02, 65Lxx, 65Mxx]—*Theory and numerics of ordinary and partial differential equations*, by M. Ainsworth, J. Levesley, W. A. Light and M. Marletta, Oxford University Press, Oxford, 1995, xiii + 333 pp., 24 cm, \$62.00

This book, the fourth in the series Advances in Numerical Analysis, consists of lecture notes by six invited speakers at the SERC Summer School in Numerical Analysis that was held at the University of Leicester in July of 1994. The topics presented fall into two categories: ordinary and partial differential equations. The stated aim of the lectures is "to be accessible to beginning graduate students and to progress to a point where, by the final lectures, current research problems could be described". In my opinion the lectures have succeeded admirably in this regard. Of particular note is the effort to make ancillary materials available to the reader. For example, Professor Corliss provides the reader with an anonymous ftp address for the LaTeX source file, MAPLE worksheets, and bibliography used in his lecture notes. Professor Johnson gives an anonymous ftp site and a WWW URL for various versions of the Femlab software package used in solving initial/boundary problems for ODE's and PDE's. Similarly, Professor Petzold provides electronic Internet references (via Netlib) to software for solving ODE's and DAE's.

In the area of ordinary differential equations the speakers and the title of their lectures are as follows:

1. George Corliss: *Guaranteed Error Bounds for Ordinary Differential Equations*, pp. 1–75.
2. Linda Petzold: *Numerical Solution of Differential-Algebraic Equations*, pp. 123–142.
3. Marino Zennaro: *Delay Differential Equations: Theory and Numerics*, pp. 291–333.

The lectures in partial differential equations are:

1. Claes Johnson (with Kenneth Eriksson, Don Estep, and Peter Hansbo): *Introduction to Computational Methods for Differential Equations*, pp. 77–122.

2. Ian Sloan: *Boundary Element Methods*, pp. 143–180.

3. Andrew Stuart: *Perturbation Theory for Infinite Dimensional Dynamical Systems*, pp. 181–290.

In Professor Corliss's lectures the author develops many of the ideas and techniques of interval analysis as they apply to initial-value problems for ODE's. He discusses the notion of a validated solution, the cost involved, and three techniques used in self-validating algorithms: (i) use of differential inequalities (ii) defect controlled solutions (iii) Lohner's method. Of these three methods the last is most practical and most general.

Professor Petzold provides a brief introduction to numerical methods, Runge-Kutta and multistep, for systems of the form $F(t, y, y') = 0$. For each of these classes of numerical method she provides a general convergence theorem and a discussion of the available software, most notably DASSL and its extensions.

In his presentation of DDE's, Professor Zennaro provides a nice introduction to the theory of delay differential equations as well as Runge-Kutta methods (theory and practice) for the solution of such problems. Considerable attention is paid to the stability properties of the methods introduced.

The adaptive numerical solution of differential equations using Galerkin methods with piecewise polynomial trial space, i.e. finite element methods, is the subject of Professor Johnson's article. As is pointed out in the section entitled Concluding remarks, the approach presented in these notes is quite different than the classical treatment of numerical methods for differential equations. The essential building blocks are a posteriori error estimates and, to a lesser extent, a priori error bounds. A particularly appealing aspect of this work is the basic uniform methodology for elliptic as well as time-dependent parabolic and hyperbolic PDE's. Much of this material is relatively new having been developed in the last several years.

Boundary element methods start with an integral (over the boundary) formulation of boundary value problems. By discretizing the boundary and defining corresponding piecewise polynomial spaces one obtains a discrete problem that is smaller than that given by the finite element method. The resulting matrix problem is dense and small as opposed to the FEM where one gets large and sparse matrices. In his lecture, Professor Sloan gives a concise survey of both Galerkin and collocation techniques as applied to boundary integral (single or double layer) equations.

The interpretation of data obtained from numerical procedures for initial value problems over *long intervals* of time is addressed in Professor Stuart's article. The framework for this study is in that of an abstract evolution equation acting in a Hilbert space. The main theme of the lecture is the study of the effect of perturbations on certain invariant (under the evolution equation) sets. This rather lengthy article is primarily theoretical in nature with remarks to applications such as the Cahn-Hilliard equation.

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18[41-00]—*Selected topics in approximation and computation*, by Marek A. Kowalski, Krszysztof A. Sikorski and Frank Stenger, Oxford University Press, Oxford, 1995, xiv + 349 pp., 24 cm, $65.00

This monograph presents results of the three authors' scientific research in their broader approximation theoretical context. The selected topics indicated in the title are approximation by Sinc functions, moment problems, n-widths, and s-numbers. The book is intended as a self-contained introduction for researchers and students in approximation theory, providing a total of 180 exercises to deepen the reader's understanding of the subject matter.

The text is organized in 8 chapters, with a preface at the beginning, and an index at the end of the book. Each chapter consists of several sections, the last of which are devoted to annotations, historical notes and references. Each section in turn contains subsections as well as the relevant exercises.

Chapter 1 provides the necessary basic material in classical approximation, both in inner product spaces and in the uniform norm. A short introduction to polynomial and general splines is given in Chapter 2. As a centerpiece of the monograph, approximation by Sinc and Sinc-like methods is studied in Chapters 3 and 4, including derivatives, integrals and convolutions of Sinc functions. Chapter 5 deals with moment problems, again including Sinc functions, while Chapter 6 is concerned with n-widths and s-numbers. Complexity questions are the issue for Chapter 7, in which a theory of optimal computational methods for nonlinear approximation problems is developed. Finally, in Chapter 8, some applications are treated, namely Sinc solutions of Burgers' equation, relations of n-widths to signal recovery, and a nonlinear zero finding problem for smooth functions.

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