MATHEMATICS OF COMPUTATION
Volume 72, Number 243, Pages 1573–1576
S 0025-5718(03)01604-1
Article electronically published on March 27, 2003

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 2000 Mathematics Subject Classification can be found in print starting with the 1999 annual index of *Mathematical Reviews*. The classifications are also accessible from `www.ams.org/msc/`.

3[60-01, 65C40, 65-01]—*Finite Markov chains and algorithmic applications*, by O. Häggström, London Mathematical Society Student Texts, Cambridge University Press, Cambridge, 2002, vol. 52, x+114 pp., hardcover, \$60.00; softcover, \$21.00

The book under review is an undergraduate textbook on finite Markov chains and algorithms such as MCMC, Propp–Wilson and simulated annealing methods. The text is based upon a course that the author has given at Chalmers University, which explains why the presentation is particularly pedagogic.

Chapter 1 and 2 revisit the very basic results in probability theory and on finite space valued Markov chains which are necessary and sufficient to understand the rest of the book.

Chapter 3 explains simulation techniques for finite Markov chains. It contains pertinent recommendations on the choice of pseudorandom number generators.

Chapters 4 and 5 deal with the advanced notions of irreducibility, aperiodicity and ergodicity. The author succeeds in making these notions easy to understand for undergraduate students without losing any mathematical rigour. In particular, the book offers a nice exposition of the coupling method to prove the convergence of the distribution at time n of an ergodic chain towards the stationary distribution.

Chapter 6 gives some notion on reversible Markov chains and presents a few classical examples of such chains.

Chapter 7 explains how to simulate a given probability measure Π by means of MCMC methods, that is, how to simulate a nonstationary ergodic Markov chain whose Π is the limit distribution. The author first explains how to solve the hard-core problem, and then enlarges the discussion by providing basic notions on Gibbs samplers, particularly the Metropolis method.

Chapter 8 is devoted to the convergence rate of the Gibbs samplers.

Chapter 9 presents applications of MCMC methods to counting problems, especially randomized polynomial time approximation schemes for counting colorings for graphs.

Chapter 10 introduces a refinement of the MCMC method, that is, the Propp–Wilson algorithm. Its termination in a finite number of steps is discussed. Simplifications of the Propp–Wilson algorithm are shown to be incorrect.

Chapters 11 and 12 explain the sandwiching technique to make the Propp–Wilson method work on large state spaces, and techniques to avoid which store random sequences during the implementation of the method.

Chapter 13 gives some elements on the simulated annealing method.

Chapter 14 concludes the book by comments on the (somewhat too short) bibliography.

The book is short but dense. The redaction is extremely elegant and living. The numerous examples perfectly well illustrate the more theoretical points. I am sure that students will find great pleasure in using the book—and that teachers will have the same pleasure in using it to prepare a course on the subject. Consequently, the reviewer would appreciate that the author write a second book which would be more advanced and designed for graduate students and researchers.

Denis Talay
E-mail address: denis.talay@sophia.inria.fr

4[65F22, 47A52, 65J20, 65N21, 65C60, 35R30, 68U10]—*Computational methods for inverse problems*, by Curtis R. Vogel, Frontiers in Applied Mathematics, SIAM, Philadelphia, PA, 2002, xvi+183 pp., hardcover, \$56.00

Inverse problems arise in many industrial and scientific applications, which caused a considerable increase of attention in the applied mathematics community over the last decades. Several monographs on this subject have been published within the last decade, focusing either on theoretical and regularization aspects of ill-posed inverse problems or on rather specific types of problems such as inverse scattering or parameter estimation. This book addresses another important aspect of inverse problems, namely their computational solution, in particular in situations enforcing more advanced methods than just those from numerical linear algebra.

The book contains nine chapters, where the first three give a basic introduction to inverse problems and review techniques of regularization and numerical optimization. Readers familiar with these subjects should be able to skip ahead and continue reading the following chapters without difficulty in understanding. However, these chapters provide a nice review of known techniques, and should be useful in particular when the book is used as a text book for a course on the subject. Chapter 4 gives an introduction to statistical estimation theory, a subject often ignored in inverse problems textbooks, but important for understanding statistical motivations for regularization methods and the implications of different noise models. Moreover, the EM algorithm is introduced in this chapter and its behavior is illustrated in a linear example.

The remaining five chapters discuss computational methods for specific types of inverse problems, which enforce a special numerical treatment. Chapter 5 is devoted to the important field of image deblurring, including fft methods and preconditioning techniques for the arising Toeplitz and block Toeplitz systems. Chapter 6 discusses iterative methods for parameter identification, with particular emphasis on the efficient computation and approximation of gradients and of Hessians, which is important in order to obtain computational methods with reasonable effort. Chapter 7 discusses some deterministic and stochastic parameter selection methods and their implementation, an important task for practical applications. Chapter 8 is devoted to computational methods for total variation regularization ranging from fixed-point type methods to primal-dual Newton methods. The behavior and convergence speed of the different methods are illustrated and compared in simple test problems, which help the reader to gain further insight. The final chapter 9 discusses numerical methods for problems with nonnegativity constraints, which appear frequently in applications. Besides methods based on projection, this part also discusses the frequently used Richardson–Lucy iteration as a special case of the EM algorithm.

This book can be used very well as a text book for a course on the subject, each chapter even contains lots of interesting exercises. It is also a good book for anyone interested in getting an introduction to the computational problems and methods arising in inverse problems. For researchers in this field, it might develop into a standard reference source.

Martin Burger
Industrial Mathematics Institute
Johannes Kepler University

5[65-02, 65N30, 65N15, 65N12]—*The finite element method for elliptic problems*, by P. G. Ciarlet, Classics in Applied Mathematics, SIAM, Philadelphia, PA, 2002, vol. 40, xxiv+530 pp., softcover, $77.00

The finite element method, in its many variants, had been firmly established in the 1960's as a general-purpose practical method for solving various partial differential equation problems of engineering, and other areas of interest, in particular for problems involving complicated geometries; cf., e.g., Zienkiewicz [1].

In the 1970's there followed a cornucopia of books devoted to setting the method on a firm mathematical footing. As examples I mention Aubin [2], where the Aubin–Oganesjan–Nitsche duality "trick" can be found on page 11; Babuska and Aziz [3], which addressed a plethora of practical issues still with us; Strang and Fix [4], which has a freshness that still makes it exciting reading and, in my opinion, has one of the very best expositions of the finite element method in eigenvalue problems; and Prenter [5], which, although later than the books mentioned above, did not include the Bramble–Hilbert lemma and thus, after a series of approximation results proved by Taylor expansions on rectangular grids, on page 127, exclaims, "Waning sadism forbids us to go further."

The book briefly under review appeared in 1978 (reprinted 2002). It well deserves to be reprinted as a classic. It is a treatise in the best tradition which combines exactness with lucidity. For example, "finite elements" are very precisely defined as triples (K, P, Σ), see page 78, but this basic definition is not introduced until after a thorough discussion of examples. It elucidates almost all of the basics of mathematical finite element analysis known at the time.

To conclude this review, I shall not go into detail about what has aged well and what has not; neither shall I compare it with later books on finite elements. Let me just make two remarks.

First, the most serious competitor to this book, at the level and detail involved, is written by Ciarlet himself, Ciarlet [6]; a 320 page article. In his own words, "This article is a revised, updated and enlarged edition of these parts of my book, 'The Finite Element Method for Elliptic Problems' [...] that are relevant here. Although I have added more than 230 items to the 316 references that I have kept from this book, I have made no attempt to compile an exhaustive bibliography."

Second, one minor thing that has not aged well is, perhaps, the general tenor of using, in the basic approximation theory, only interpolants for functions that have point-values (or, even point-values for derivatives). This imposes unnecessary

Sobolev type restrictions on the theory and is misleading to a casual reader. (Of course, Ciarlet was well aware of this, see the third paragraph on page 170.)

All in all, a very welcome reprinted classic!

References

1. O.C. Zienkiewicz, *The finite element method in engineering science*, McGraw-Hill, London, 1971. MR **47:**4518
2. Jean-Pierre Aubin, *Approximation of elliptic boundary-value problems*, Pure and Applied Mathematics, Vol. XXVI. Wiley-Interscience, 1972. MR **57:**18139
3. I. Babuska and A.K. Aziz, *Survey lectures on the mathematical foundations of the finite element method*, in: The Mathematical Foundations of the Finite Element Method with Applications to Partial Differential Equations (A.K. Aziz, Ed.), pp. 3–359, Academic Press, New York, 1972. MR **54:**9111.
4. Gilbert Strang and George J. Fix, *An analysis of the finite element method*, Prentice-Hall, Englewood Cliffs, NJ, 1973. MR **56:**1747.
5. P.M. Prenter, *Splines and variational methods*, John Wiley and Sons, New York, 1975. MR **58:**3287
6. P.G. Ciarlet, *Basic error estimates for elliptic problems*, in: Handbook of Numerical Analysis, Volume II, Finite Element Methods, Part 1 (P.G. Ciarlet and J.L. Lions, Eds.), pp. 17-352, North-Holland Amsterdam, 1991. MR **91f:**61005

Lars B. Wahlbin