CORRIGENDUM TO "AN INFINITE FAMILY OF BOUNDS ON ZEROS OF ANALYTIC FUNCTIONS AND RELATIONSHIP TO SMALE'S BOUND"

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In [1], Theorem 3.2 proves the following lower bound on the gap between two zeros ξ and ξ' of a complex-valued function $f(z)$, that is analytic everywhere on the complex plane, where ξ is assumed to be a simple root:

$$(1) \qquad |\xi - \xi'| \geq \frac{r_m}{\gamma_m(\xi)}.$$

In (1) r_m is some constant and $\gamma_m(\xi)$ is a certain function of the normalized derivatives of f at ξ (see (3.1) in [1]).

While Theorem 3.2 is valid, the present proof in [1] is not. The quantity u in the proof should actually be $|\xi - \xi'|\gamma_m(\xi')$. But this implies that the present proof in [1] actually establishes the inequality

$$(2) \qquad |\xi - \xi'| \geq \frac{r_m}{\gamma_m(\xi')}.$$

If we assume that ξ' is also a simple root, then clearly a symmetry argument also proves the inequality (1) and, hence, the present proof remains valid. But this added assumption is unnecessary.

To correct the proof of Theorem 3.2 essentially all that is needed is to switch ξ and ξ'; i.e., assume that ξ' is a simple root while relaxing the simplicity assumption on ξ.

Except possibly for a finite set of values of z, the following expansion formula, given as equation (2.7) in [1], is valid even when ξ is not a simple root of $f(z)$:

$$(3) \qquad B_m(z) = \xi + \sum_{k=m}^{\infty} (-1)^m \frac{\widehat{D}_{m-1,k}(z)}{D_{m-1}(z)} (\xi - z)^k.$$

Thus, the assumption of simplicity of ξ in Proposition 3.1 of [1] is also superfluous. Then the present proof of Theorem 3.2 in [1] applies and establishes the inequality in (2) where ξ' is assumed to be a simple root of $f(z)$. But this is equivalent to the statement of Theorem 3.2 in [1] with the role of ξ and ξ' interchanged.

References

[1] B. Kalantari, *An infinite family of bounds on zeros of analytic functions and relationship to Smale's bound*, Math. Comp., **74** (2005), 841–852. MR2114651

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