MATHEMATICS OF COMPUTATION
Volume 76, Number 259, July 2007, Pages 1693–1694
S 0025-5718(07)02023-6
Article electronically published on March 9, 2007

3[65Nxx, 65Dxx, 65Lxx, 65Z05, 35Jxx, 35Lxx, 76M10]—*Partial differential equations and the finite element method*, by Pavel Solin, Wiley Interscience, Hoboken, NJ, 2006, xxvii+472 pp., hardcover, US$105.00, ISBN 0-471-72070-4

This is a textbook for upper-level undergraduate students and graduate students of computational engineering and science. It also serves as a practical problem solving reference for researchers, engineers, and physicists. The main body consists of four parts: Chapter 1, partial differential equations; Chapters 2-4, finite element method; Chapter 5, numerical solution of ordinary differential equations; and Chapters 6 and 7, the application of the finite element method to elasticity and electro-magnetism.

Some basic definitions, concepts, and theorems of the partial differential equations are selected in Chapter 1. They are: classification of second order equations, well-posedness, weak formulation, Lax-Milgram lemma, unique solvability of some typical boundary value problems and initial-boundary value problems, maximum principle, first order hyperbolic systems of equations. Most of these are applied in the following chapters.

Starting from one-dimensional problems, the author studies the theory and implementation of the finite element method in Chapters 2–4. The Galerkin method, triangulation of the underlying domains, shape functions, interpolation, assembling of elements, and data structures are examined in detail. A variety of one-dimensional and two-dimensional Lagrange elements are considered. Special emphasis is put on higher-order elements. Therefore, numerical quadrature and orthogonal systems of polynomials play important roles in the content. In this respect, the author discusses the Gauss quadrature, the Legendre polynomials, and the Chebyshev, Gauss-Lobatto and Fekete nodal points. This kind of approach can be specified as the spectral-element method.

The evolution partial differential equations are reduced to systems of ordinary differential equations by a semi-discretization. Chapter 5 is about the numerical schemes for the initial value problems of ordinary differential equations. Several schemes are studied, such as the explicit and implicit Euler scheme, the explicit and implicit Runge-Kutta scheme. Moreover, stability of schemes is defined and analyzed .

Elasticity problems are dealt with in Chapter 6. The weak forms of different models are defined and analyzed. They are: the Euler-Bernoulli model, the Reissner-Mindlin plate model, and the Kirchhoff plate model. The material of finite elements is also presented for one-dimensional problems at first. Hermite elements are studied. For two-dimensional problems some triangle Hermite elements with third or higher order are presented, which are nonconforming. Argyris elements are also presented, which are conforming. Special emphasis is also put on the quadrature schemes and interpolating nodes.

Maxwell equations with boundary conditions are presented in detail in Chapter 7. The potential formulation, the time-harmonic Maxwell equations, and the Helmholtz equation are also presented. The Fredholm alternative theorem is applied to study existence and uniqueness of a solution. Edge elements are introduced to the finite element method of these equations. The elements include: Whitney elements and Nedelec elements.

Mathematical analysis in this book is mainly on the well-posedness of a number of variational problems, and the conformity of a number of elements. The analysis can be found in most chapters. On the other hand, the analysis of convergence and error estimates is not the core ingredient of the book. Convergence is proved for one-dimensional elliptic equations, while no convergence analysis is available for the other problems.

Notations and terminologies of functional analysis are employed throughout all chapters. To provide the fundamental and elementary knowledge, Appendix A contains a brief introduction of functional analysis. Definitions and properties of linear spaces, linear operators, Banach spaces, Hilbert spaces, and Sobolev spaces are presented, which are necessary for understanding the theory in this book. In Appendix B , the introduction of the sMatrix utility is given, and an efficient way for connecting the packages, PETs, Trilinos and UMFPACK, to a finite element solver is presented. The author also gives a brief description of the high performance modular finite element system HERMES. Five examples are given to show the implementation of the finite element method to an engineering or scientific problem.

In my opinion, this is a well-written and systematic book. All materials were selected carefully. A number of illustrative examples and algorithms are shown to help the readers. There are some exercise problems attached to the text. The entire book is readable and practicable. In general, students may find it difficult to read literature on the finite element method by lacking the necessary knowledge of functional analysis and partial differential equations. This book provides the necessary materials. The amount is appropriate and prerequisites are elementary.

Finally, the author is interested in higher-order elements, and I believe both lower-order elements and higher-order elements possess their own advantages and disadvantages. Nevertheless, this book shows the advantages of higher-order elements, which are left for the reader's consideration.

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