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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme accessible from www.ams.org/msc/.

1[65-01, 65Lxx, 65Mxx, 35Kxx, 35Lxx]—*Numerical methods for evolutionary differential equations*, by Uri M. Ascher, SIAM, Philadelphia, PA, 2008, xiv+395 pp., softcover, US $79.00, ISBN 978-0-898716-52-8

The numerical treatment of evolutionary (ordinary and partial) differential equations is a timeless topic and it has occupied numerical analysts for many decades. There are interesting applications throughout the sciences, ranging from physics, astronomy, biology, chemistry, earth sciences to image processing, computer vision, and mathematical finance. Due to their variety and importance, there are many books and publications devoted to this topic treating the modeling of physical phenomena, mathematical techniques for a deeper theoretical understanding, and numerical algorithms for an efficient and reliable computer simulation.

The monograph of Uri Ascher is in many respects different from others on this topic. It emphasizes the essential ideas, and studies principles, properties, and usage of numerical methods from a point of view of general applicability. At the same time, it avoids heavy notation, difficult proofs, and technical details, but gives suitable suggestions to literature for additional reading. This makes it possible to cover in about 390 pages a wide range of interesting topics in the field of evolutionary differential equations.

Intended for a broad audience of scientists and students, the book assumes only the basic knowledge in calculus, linear algebra, and differential equations. It is therefore not only accessible to mathematicians and physicists but also to engineers and students from other disciplines who want to know more about numerically solving differential equations.

The main topic of the monograph is numerical methods for partial (parabolic and hyperbolic) differential equations, but it also treats relevant topics on the numerical solution of ordinary differential equations. The introductory Chapter 1 gives a flavor of simple model problems (advection-diffusion, heat equation), discusses their well-posedness, and presents a taste of finite differences. It reviews matrix norms, function spaces, and the Fourier transform. Chapter 2 is the first (out of two) that is solely devoted to ordinary differential equations. Linear multistep and Runge–Kutta methods are introduced, and their stability and convergence are discussed. It also addresses the problem of stiffness and that of highly oscillatory solutions. Semi- and full-discretizations are the subject of Chapter 3. Emphasis is put on finite differences, but also finite volume and finite element methods are outlined. Stability concepts are essential for designing and analyzing numerical methods for time-dependent problems. Chapter 4 is concerned with constant coefficient problems (Fourier analysis and eigenvalue analysis), whereas Chapter 5 considers variable coefficients and nonlinear problems.

More advanced topics are treated in the rest of the monograph. Chapter 6 (the second one devoted to ordinary differential equations) gives a glimpse at geometric numerical integration, a highly actual topic that focuses on qualitative features of numerical solutions over long time intervals. Symplectic integrators (Runge–Kutta, splitting, composition, and variational methods) for Hamiltonian systems are presented and their long-time behavior is discussed. Chapter 7 introduces the concept of numerical dispersion, discusses Hamiltonian partial differential equations and examines the performance of multi-symplectic integration schemes. Chapter 8 is on handling boundary conditions, whereas Chapter 9 considers extensions to more than one space dimension, splitting methods, and iterative methods for large linear systems. Partial differential equations that can be written in the form of a conservation law, are the subject of Chapter 10. Godunov's scheme, ENO (essentially nonoscillatory) and WENO schemes, and strong stability preserving methods are discussed. The final Chapter 11 briefly evokes the connection between optimization and differential equations, the use of nonuniform meshes, and level set methods.

It is an attractive feature of the monograph that most concepts are motivated and developed by examples. Each individual chapter (with exception of the last one) contains a section of exercises and review questions, which makes this book also suitable for self-study. Having said all this, one can conclude that the monograph by Uri Ascher is very useful for an elegant and quick introduction to the numerical treatment of evolutionary differential equations.

Ernst Hairer
Université de Genève, Switzerland
E-mail address: Ernst.Hairer@unige.ch