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3[65K05, 90C48, 65K10]—*Lagrange multiplier approach to variational problems and applications*, Kazufumi Ito and Karl Kunisch (Editors), SIAM, Philadelphia, PA, 2008, xviii+341 pp., softcover, US$99.00, ISBN 978-0898716-49-8

The book under consideration deals with a general class of nonlinear variational problems of the form

$$\begin{aligned} &\min_{y\in Y, u\in U} f(y,u), \\ &e(y,u)=0,\ g(y,u)\in K, \end{aligned} \tag{0.1}$$

where $Y,\ U,\ W,\ Z$ are Banach spaces, K is a closed convex set in Z, $f : Y\times U \to \mathbb{R}$ denotes the cost functional, and $e : Y \times U \to W,\quad g : Y \times U \to Z$ are functionals describing equality and inequality constraints. A special example of K is simple box constraints $K = \{z \in Z : \phi \leq z \leq \psi\}$ with the given elements ϕ, ψ in a lattice Z with ordering $\leq$.

The first part of the book deals mainly with theoretical issues including the existence of minimizers, optimality conditions, Lagrange multiplier theory, sufficient optimality considerations as well as the sensitivity analysis of the solutions with respect to perturbations in the problem data. Some selected computational methods for solving the constraint minimization problem (0.1) are the topics of the second part. The general approach that the authors follow for the analytical as well as numerical treatment of the problem is based on Lagrange multiplier theory. This theory provides a tool for the analysis of general constrained optimization problems with not necessarily differentiable cost functionals and with state equations which can be in some sense singular. It is also a theoretical basis for developing efficient iterative methods for solving such problems.

The tools to establish existence of Lagrange multiplier as well as a variety of illustrating examples are described in Chapter 1. Chapter 2 deals with the sensitivity analysis of abstract constrained nonlinear optimization problems and contains results which are not only of theoretical but also of practical importance because they have been a starting point for the development of various algorithmic concepts for decades.

The smooth optimization problems are the main topics of Chapters 3, 5 and 6 at which Chapter 3 covers first order augmented Lagrangian methods for optimization problems with equality and inequality constraints. Inverse problems formulated as regularized least squares problems and optimal control problems for partial differential equations are typical examples of the theories discussed here. Chapter 5 is devoted to the Newton and the sequential quadratic programming (SQP) methods for iterative solving of equality-constrained problems. Chapter 6, "Augmented Lagrangian-SQP Methods", extends the collection of quadratically convergent iteration methods from the previous chapter.

Chapters 4, 7, 8 and 9 form a second large unit, which is characterized by the keywords: nonsmoothness, primal-dual active set strategy, and semismooth Newton methods. A key result of Chapter 4 is the formulation of differential inclusions which arise in optimality systems by means of nondifferentiable equations and which serve as the basis for the primal-dual active set strategy. This strategy and its global convergence properties for unilaterally and bilaterally constrained problems are the topics of Chapter 7. Chapter 8, "Semismooth Newton Methods I", contains the local analysis of the primal-dual active set strategy in the framework of semismooth Newton methods and establishes local superlinear convergence of the Newton method for problems which do not satisfy the classical sufficient conditions of local quadratic convergence. The next chapter, "Semismooth Newton Methods II: Applications", is devoted to such important classes of applications as image restoration and deconvolution problems regularized by the bounded variation (BV) functional as well as friction and contact problems in elasticity. Chapter 10 deals with a Lagrangian treatment of parabolic variational inequalities in unbounded domains (arising, for example, in the Black-Scholes equation) and describes the use of monotone techniques for analyzing parabolic systems without relying on compactness assumptions in a Gelfand-triple framework.

Finally, a calculus for obtaining the shape derivative of the cost functional in shape optimization problems which bypasses the need for using the shape derivative of the state variables of the partial differential equations is the topic of Chapter 11.

The book is written very clearly, the main text is supplemented with a large collection of examples that significantly complement the main text. The list of references containing more then 150 titles and the subject index round up the book.

The book can serve as a foundation for a one or two semesters course for students or can be useful as support for all who are studying or teaching optimization problems, their numerical treatment and applications. The researcher can find interesting references and ideas giving access to modern results in the field.

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