5005-B1-26 **Roman Vershynin*** (vershynin@math.ucdavis.edu), Department of Mathematics, University of California, Davis, One Shields Ave, Davis, CA 95616, and **Mark Rudelson** (rudelson@math.missouri.edu), Department of Mathematics, University of Missouri, Columbia, Columbia, MO 65213. *Von Neumann, random matrices and sparse approximation.*

Sparse approximation has provided a natural context for the recent solution of some basic conjectures on condition numbers of random matrices. Von Neumann and his associates predicted that the condition number of random $n \times n$ matrices A with independent entries should be $O(n)$. More precisely, the smallest singular value of A should be of order $n^{-1/2}$ with high probability. For Gaussian matrices, Smale conjectured and Edelman proved this in 1988. For ± 1 matrices, this has been an ICM 2002 conjecture of Spielman and Teng. We will discuss the solution to this problem in full generality.

Our argument is based on a decomposition of the space $\mathbb{R}^n$ into the sets of compressible and incompressible vectors (signals) and establishing different restricted isometry principles (R.I.P.) for A on each of these sets. For compressible vectors, this is the R.I.P. of Candes and Tao, which guarantees recovery of compressible signals by convex programming. For incompressible vectors, the R.I.P. rests on the development of the Littlewood-Offord theory of anti-concentration probabilistic bounds, which brings in some ergodic theory and additive combinatorics.

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