

## Titles and Abstracts

Alexei Borodin

Title: Tau-function of isomonodromy transformations of difference equations

Abstract: The goal of the talk is to introduce an analog of the Jimbo-Miwa-Ueno tau-function for linear difference equations with rational coefficients, and to explain its relation to gap probabilities in discrete random matrix type models.

Wlodek Bryc

Title: Processes with quadratic regressions

Abstract: The family of stochastic processes with uncorrelated increments, linear regressions, and quadratic conditional variances includes classical Levy processes: the Wiener, Poisson, Gamma, and Pascal processes. It includes also Markov processes whose bivariate distributions match the bivariate distributions of certain free Levy processes, including free Brownian motion and free Poisson process, as well as the classical version of the  $q$ -Brownian motion, and the classical version of the  $q$ -Poisson process. A fairly complete description of such processes is now available in terms of their orthogonal martingale polynomials.

Momar Dieng

Title:

Abstract:

Jeffrey DiFranco

Title: Asymptotics of Tracy-Widom distribution functions.

Abstract: The Tracy-Widom (TW) distribution functions appear in numerous areas of combinatorics and probability. In particular these functions are the limiting distributions of the largest eigenvalues in random matrix theory. The asymptotic expansions for  $x$  approaching negative infinity have been known except for a multiplicative constant. The constant in the GUE case, conjectured in the original work of Tracy and Widom, was obtained in the recent work of Deift, Its, and Krasovsky. We discuss an alternate proof of the constant term in the GUE case as well as the calculation of the constant in the GOE/GSE cases. Additionally, we are able to obtain a total integral formula for the Hastings-McLeod solution of the Painleve II equation. This is a joint work with Baik and Buckingham.

Torsten Ehrhardt

Title: Asymptotics of the determinants of Wiener-Hopf Hankel operators with sine-kernel.

Abstract: I will talk about the asymptotics of the determinants of  $I - K$  where  $K$  is the integral operator with the kernel

$$\frac{\sin(x-y)}{\pi(x-y)} \pm \frac{\sin(x+y)}{\pi(x+y)}$$

on the finite interval  $[0, \alpha]$  with  $\alpha \rightarrow \infty$ . These determinants appear in the level spacing distributions of the GOE and GSE as well as of the Laguerre and Jacobi ensembles in two particular cases. The proof is based on precise relations between Hankel and Toeplitz plus Hankel determinants as well as on the study of the asymptotics of Wiener-Hopf plus Hankel determinants with Fisher-Hartwig symbols.

Patrik Ferrari

Title: Transition between  $Airy_1$  and  $Airy_2$  processes and TASEP fluctuations

Abstract: Half a decade ago, Praehofer and Spohn discovered the  $Airy_2$  process in a surface growth model. It appeared to be one of the universal processes, appearing in different models including random matrix theory. The model we focus on is the totally asymmetric simple exclusion process (TASEP), which can also be seen as a growth model. In the TASEP, the  $Airy_2$  process occurs from step-like initial conditions, where the corresponding limit shape is curved. The analogue of the  $Airy_2$  process in the case of flat limit shape, has been recently unravelled and called  $Airy_1$  process. However, in a typical situation one can have coexistence

of the two processes in separated regions. Of course there is a transition region, which we recently analyzed and we determined the transition process between the *Airy*<sub>2</sub> and the *Airy*<sub>1</sub> process.

Peter Forrester

Title: Integrability and exact results for general  $\beta$

Abstract: With  $\Delta$  denoting the Vandermonde product of eigenvalue differences, classical random matrix theory gives rise to eigenvalue PDFs of the form  $\prod_{l=1}^N w(\lambda_l) |\Delta|^\beta$ , for  $w$  a classical weight function and  $\beta = 1, 2$  or  $4$ . In recent years random tridiagonal and unitary Hessenberg matrices have been identified which realize these PDFs for general  $\beta > 0$ . Although not possessing determinant and Pfaffian structures characteristic of  $\beta = 1, 2$  and  $4$ , there are nonetheless underlying integrable structures, and these lead to the exact evaluation of correlations and distribution functions, as will be explained.

Dimitri Gioev

Title: Asymptotic expansion of the partition function for orthogonal and symplectic ensembles

Abstract: Asymptotic expansions of the partition function for unitary random matrix ensembles and their connection with map enumeration problems appeared in the physics literature see Bessis-Itzykson-Zuber (1980), in particular. The rigorous proof of the complete expansion for unitary ensembles and the map enumeration relations were found by Ercolani and McLaughlin (2002), see also the works Bleher-Its (2004) and Ercolani-McLaughlin-Pierce (2006). These results use the asymptotic information for quantities related to orthogonal polynomials available from the Riemann-Hilbert analysis.

Asymptotic expansions of the partition function for multi-matrix unitary ensembles were recently obtained in the works of Guionnet and Maurel-Segala by different methods.

The leading order asymptotics of the partition function was found by Johansson (1998) in the case of an arbitrary  $\beta$ .

We will present a derivation of the complete expansion for the partition function for orthogonal and symplectic ensembles. The result holds for a small (in particular, one-cut) polynomial perturbation of the Gaussian weight. The proof is based on the combination of the methods in Ercolani-McLaughlin (2002) and in the work Deift-Gioev (2004) on universality.

John Harnad

Title: Tau functions, integrable systems and random processes

Abstract: Tau functions are essential tools in the theory of integrable systems, both classical and quantum, and also appear as partition functions, gap probabilities and spectral correlators in Random Matrix theory. They can also be seen to play a role in certain classes of random processes and statistical models. This is apparent, e.g., in the work of Okounkov, Reshetikhin, Olshansky and Borodin, on statistical weights and limiting shapes in 1 and 2 dimensional random partitions. Even Bethe ansatz techniques can be applied to "solve" certain random processes, as was shown long ago, e.g., in the work Gwa and Spohn. In this talk, I summarize some joint work with Alexander Orlov, in which new aspects of the theory of tau functions and integrable systems are used in the study of statistical systems of fermi particles and in random processes.

Eugene Kanziiper

Title: Statistics of Real Eigenvalues in GinOE spectra and a Pfaffian Integration Formula

Abstract: Integrable structure of Ginibre's Orthogonal Ensemble of random matrices is looked at through the prism of the probability " $p_{n,k}$ " to find exactly " $k$ " real eigenvalues in the spectrum of an " $n$ " by " $n$ " real asymmetric Gaussian random matrix. The exact solution for the probability function " $p_{n,k}$ " is presented, and its remarkable connection to the theory of symmetric functions is revealed. A concise proof of a Pfaffian integration formula, the key ingredient of the theory, will also be presented.

Doron Lubinsky

Title: Two new approaches to universality limits in the bulk

Abstract: We discuss two new approaches to universality limits involving orthogonal polynomials. The first is essentially a comparative asymptotic, and based on two measures having Christoffel functions with the

same asymptotic. It is sufficiently powerful to prove the following: Suppose we have a measure with support  $[-1,1]$  that is regular in the sense of Stahl and Totik. Then universality holds at each point at which the measure is absolutely continuous with positive and continuous density. In a tour de force of potential theory, Vili Totik has very recently shown that this method can be extended to regular measures on any compact set, and points of continuity may be replaced by Lebesgue points.

Our second (newer) method involves a uniqueness argument, and works for arbitrary measures with compact support. They do not need to be regular. However, the hypotheses are more implicit. This second method is still work in progress.

Edouard Maurel-Segala

Title: Unitary matrix models.

Abstract: We will talk about the limit when the dimension goes to infinity of some unitary matrix models. The main cases of interest are the study of the product of Haar measure and the study of the Itzykson-Zuber integral in a high temperature regime. Our approach is to consider the more general case of perturbation of the Haar measure for which we propose a diagrammatic explanation of the limit.

Ken McLaughlin

Title: Random matrices with source term, equilibrium measures and algebraic curves for the asymptotic analysis of Riemann-Hilbert problems

Abstract: First we will make sense of the title. Then, with the time remaining, we will consider an existence question for an algebraic curve, and (hopefully) see the main steps in the proof of existence.

Peter Miller

Title: Riemann-Hilbert Problems with Lots of Discrete Spectrum: Asymptotics and Applications

Abstract: I will discuss several situations in which an asymptotic limit of interest leads one to consider the construction of a matrix-valued meromorphic function with principal part data specified at asymptotically many poles. Applications include semiclassical asymptotics of integrable nonlinear wave problems (KdV, NLS, sine-Gordon) as well as statistical combinatorics (discrete orthogonal polynomial ensembles, i.e. discrete analogues of random matrix theory).

Irina Nenciu

Title: CMV matrices and the Ablowitz-Ladik hierarchy: a multi-Hamiltonian structure and the connection to the Toda lattice

Abstract: In this talk we will describe various properties of a completely integrable system, the Ablowitz-Ladik (AL) equation, through its connection to the theory of orthogonal polynomials on the unit circle. In particular, we will concentrate on its associated Poisson structure using a functional analytic approach; this in turn will allow us to define a multi-Hamiltonian structure for the AL hierarchy. As an important consequence, we show that, in some of the new Poisson brackets, the classical map taking orthogonal polynomials on the unit circle into orthogonal polynomials on the real line becomes a symplectic mapping taking certain of the AL flows into flows from the Toda lattice hierarchy.

Sandrine Peche

Title: universality results for the largest eigenvalue of sample covariance matrices

Abstract: we consider  $XX^*$ , where  $X$  is a  $N \times p$  random matrix with i.i.d. entries. We assume that these entries are centered, symmetrically distributed and have sub-Gaussian tails. Then we prove that the largest eigenvalues exhibit Tracy Widom fluctuations if  $p/N \rightarrow \gamma \in [0, \infty]$ , extending a previous universality result of A. Soshnikov.

Virgil Pierce

Title: Riemann-Hilbert problems for skew-orthogonal polynomials.

We find a local  $(d+1) \times (d+1)$  Riemann-Hilbert problem characterizing the skew-orthogonal polynomials associated to the partition function of orthogonal ensembles of random matrices with a potential function

of degree  $d$ . The partition function may be expressed directly in terms of these polynomials or can be described as the tau-function of the Pfaff lattice hierarchy. The Pfaff lattice hierarchy gives the evolution of the recursion relation for the skew-orthogonal polynomials under polynomial deformations of the potential. Our approach does not produce a Riemann-Hilbert problem for the skew-orthogonal polynomials associated to the symplectic ensembles.

Ross Pinsky

Title: The Law of Large Numbers for Increasing Subsequences in Random Permutations

Abstract: Let the random variable  $Z_{n,k}$  denote the number of increasing subsequences of length  $k$  in a random permutation from  $S_n$ , the symmetric group of permutations of  $\{1, \dots, n\}$ . We show that the weak law of large numbers holds for  $Z_{n,k_n}$  if  $k_n = o(n^{\frac{2}{5}})$ ; that is,

$$\lim_{n \rightarrow \infty} \frac{Z_{n,k_n}}{EZ_{n,k_n}} = 1, \text{ in probability.}$$

The proof uses the second moment method and demonstrates that this method cannot work if the condition  $k_n = o(n^{\frac{2}{5}})$  does not hold. It follows from results concerning the longest increasing subsequence of a random permutation that the law of large numbers cannot hold for  $Z_{n,k_n}$  if  $k_n \geq cn^{\frac{1}{2}}$ , with  $c > 2$ . Presumably there is a critical exponent  $l_0$  such that the law of large numbers holds if  $k_n = O(n^l)$ , for some  $l < l_0$ , and does not hold if  $\limsup_{n \rightarrow \infty} \frac{k_n}{n^l} > 0$ , for some  $l > l_0$ . Several phase transitions concerning increasing subsequences occur at  $l = \frac{1}{2}$ , and these would suggest that  $l_0 = \frac{1}{2}$ . However, we show that the law of large numbers fails for  $Z_{n,k_n}$  if  $\limsup_{n \rightarrow \infty} \frac{k_n}{n^{\frac{4}{9}}} = \infty$ . Thus the critical exponent, if it exists, must satisfy  $l_0 \in [\frac{2}{5}, \frac{4}{9}]$ .

Brian Rider

Title: A diffusion view of the spectral edge

Abstract: We prove the conjecture of Edelman-Sutton: the general Tracy-Widom laws for the largest eigenvalues of the RMT beta ensemble are shared by those at the bottom of the spectrum of a certain one-dimensional random Schroedinger family. The classical Riccati map then gives a second description of these distributions in terms of the explosion probability of a simple 1-d diffusion process. As for applications, determining the tail shape of the "beta Tracy-Widom" laws as well as establishing small deviations for the largest eigenvalue at all beta and finite  $n$  now follow easily.

Christopher Sinclair

Title: Correlation Functions for Ensembles of Real Asymmetric Matrices

Abstract: We give a closed form for the correlation functions for ensembles of real asymmetric matrices. As with other  $\beta = 1$  ensembles the correlation functions can be written as Pfaffians of antisymmetric matrices expressed in terms of a matrix kernel. This is joint work with Alexei Borodin.

Herbert Spohn

Title: Determinantal processes, what is beyond?

Abstract: I review some models which are analyzed through determinantal processes and perturbations thereof. I point out that there are obviously related models which are not determinantal and merely pose the question whether our mathematical machinery can be developed to handle also such cases.

Eugene Strahov

Title: A differential model for deformed Plancherel measures.

Abstract:  $q$ -deformations of the Plancherel measures originate in different contents, in particular they are closely connected with non-uniform random permutations.

We show that  $q$ -deformations of the Plancherel transition probabilities lead to deformations of the correspondence between continual diagrams and probability distributions studied by S. V. Kerov (the Markov-Krein correspondence).

This enables us to construct a differential model for deformed Plancherel measures, and to derive differential equations describing the dynamics of continual diagrams.

We find an automodel solution of these differential equations, and prove that this automodel solution is stable (i.e. it attracts all other solutions).

Craig Tracy

Title: Integral Formulas for the Asymmetric Simple Exclusion Process I

Balint Virag

Title: Hyperbolic Brownian motion and the sine point process

Abstract: The bulk point process limit of general beta matrix ensembles can be expressed as a simple functional of Brownian motion in the hyperbolic plane. This is joint work with Benedek Valko.

Van Vu

Title: From smooth analysis to circular law: A journey via additive combinatorics

Abstract: It was observed in the 1950s that the eigenvalues of a (non-Hermitian) random matrix seem to distribute uniformly in a circle (circular law). A rigorous explanation was given by Ginibre/Mehta (60s) for the Gaussian case, and by Bai (1997, following a work of Girko from 1984) for general continuous models. Bai's proof, however, does not extend to the discrete models.

Another observation in a completely different area: linear programming. It is a very well known phenomenon in the programming community that the simplex method, while proven to be exponential in the worst case, usually runs very fast, beating all other algorithms, even those are proven to be polynomial in all cases. Few years ago, Spielman and Teng came up with a nice explanation (smooth analysis) which relies on the existence of noise in the calculation process. Their analysis assumed continuous noise. The proof for the discrete (and more natural) case has been missing.

Surprising it may sound, but the two problems are strongly related !!

I will discuss recent developments which fill these missing parts, using tools from additive combinatorics, in particular the so-called Inverse Littlewood-Offord theorem.

(joint work with T. Tao, UCLA)

Harold Widom

Title: Integral Formulas for the Asymmetric Simple Exclusion Process II

Lauren Williams

Title: Tableaux combinatorics for the asymmetric exclusion process

Abstract: The partially asymmetric exclusion process (PASEP) is an important model from statistical mechanics which describes a system of interacting particles hopping left and right on a one-dimensional lattice of  $N$  sites. It is partially asymmetric in the sense that the probability of hopping left is  $q$  times the probability of hopping right. Additionally, particles may enter from the left with probability  $\alpha$  and exit to the right with probability  $\beta$ . It has been observed that the (unique) stationary distribution of the PASEP has remarkable connections to combinatorics. We will describe how in fact the (normalized) probability of being in a particular state of the PASEP can be viewed as a certain weight generating function for the permutation tableaux (certain tableaux that come indirectly from the totally non-negative part of the Grassmannian) of a fixed shape. Our first proof is algebraic and uses the matrix ansatz of Derrida et al. Our second proof involves defining a Markov chain – which we call the PT chain – on the set of permutation tableaux which projects to the PASEP in a very strong sense, thus revealing a hidden structure behind the PASEP. Via the bijection from permutation tableaux to permutations, the PT chain can also be viewed as a Markov chain on the symmetric group. Another nice feature of the PT chain is that it possesses a certain symmetry which extends the particle-hole symmetry of the PASEP. This is joint work with Sylvie Corteel.