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William K Allard* (wka@math.duke.edu), Mathematics Department, Duke University, Box 90320, Durham, NC 27707. *Examples of minimizers for total variation regularization.*

Let $\mathcal{F}(\mathbb{R}^2)$ be the family of bounded nonnegative measurable functions on \mathbb{R}^2 . Suppose $\gamma : \mathbb{R} \rightarrow [0, \infty]$ is convex and zero at zero and $s \in \mathcal{F}(\mathbb{R}^2)$. Let

$$F(f) = \mathbf{TV}(f) + \int \gamma(f(x) - s(x)) d\mathcal{L}^2x \quad \text{for } f \in \mathcal{F}(\mathbb{R}^2).$$

Such a functional F could be called a *fidelity* in that it measures how much f deviates from s .

Now suppose $0 < \epsilon < \infty$ and let

$$F_\epsilon(f) = \epsilon \mathbf{TV}(f) + F(f) \quad \text{for } f \in \mathcal{F}(\mathbb{R}^2).$$

F_ϵ could be called a *total variation regularization of F* . Minimizers of F_ϵ are used to denoise the source image s .

We describe in detail the family of minimizers for F_ϵ when s is the indicator function of a compact convex subset of \mathbb{R}^2 , or of $([0, 1] \times [0, -1]) \cup ([-1, 0] \times [0, 1])$ or of the union of two nonoverlapping circles of the same radius. It will turn out that these minimizers vary greatly as γ varies.

We believe these examples shed much light on the nature of total variation regularization. (Received January 27, 2008)