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Felipe Monroy-Pérez* (fmp@correo.azc.uam.mx), Av. san Pablo 180, Azcapotzalco, 07300 México D.F., Mexico. *Nilpotent sub-Riemannian geometry*. Preliminary report.

A step- m nilpotent sub-Riemannian geometry of type (k, n) , consists of a smooth manifold of dimension n , together with a bracket generating distribution Δ of k smooth vector fields with $k < n$, and such that all the non-zero commutators have length at most m . In this paper we study the case when each vector field X_j in Δ is written in local coordinates $(x, t) = (x_1, \dots, x_{n-1}, t)$ as follows

$$X_j = \frac{\partial}{\partial x_j} + \xi^j(x) \frac{\partial}{\partial t},$$

with $\xi^j(x)$ a polynomial. In this case, the Lie algebra \mathfrak{g} generated by Δ is finite-dimensional, nilpotent and its nilpotency depends on $\max\{\deg \xi^i(x)\}$. We calculate the dimension of \mathfrak{g} and describe explicitly a Philip Hall basis for it. We also discuss the sub-Riemannian geodesic problem associated to Δ , i.e., the minimization of the length functional

$$\ell(g) = \int \sqrt{\langle \dot{g}, \dot{g} \rangle}$$

in the class of horizontal curves, that is, curves $\tau \mapsto g(\tau)$, satisfying $\dot{g}(\tau) \in \Delta(g(\tau))$ a.e. We show that some low dimensional cases can be explicitly integrated by means of elliptic functions. (Received January 29, 2008)