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Joseph Gubeladze* (soso@math.sfsu.edu), Department of Mathematics, San Francisco State University, San Francisco, CA 94132. *Commutative algebra of lattice polytopes with long edges.*

By homogenization, a lattice polytope $P \subset \mathbb{R}^n$ gives rise to an affine submonoid $M(P) \subset \mathbb{Z}^{n+1}$. For a field k , the monoid ring $k[M(P)]$ is graded in a natural way. The homological properties of $k[M(P)]$, captured by the minimal graded free resolution of k over $k[M(P)]$, have been studied a lot in combinatorial commutative algebra. For instance, if one scales P by a factor $c \in \mathbb{N}$ with $c \geq \dim P$, the resulting ring $k[M(cP)]$ becomes Koszul. When, instead of homothetically blowing up the ground polytope, one considers lattice polytopes whose edges contain sufficiently many lattice points, depending on $\dim P$, similar results turn out to be surprisingly difficult to prove. Even the 0th homological slice of the Koszul property, the normality property, had been an open question for some time. It was answered in the positive only recently. In the talk I will also explain why one may expect that the analogous claim on the next homological level is true as well – that the toric ring $k[M(P)]$ is defined quadratics, assuming P has ‘long’ edges with respect to $\dim P$. This leads to an obvious general conjecture whose proof, unfortunately, seems well beyond the currently available technics. (Received April 12, 2010)