Meeting: 998, Houston, Texas, SS 1A, Special Session on Graph Theory and Combinatorics

998-05-386 Etienne De Klerk (edeklerk@math.uwaterloo.ca), John Maharry
(maharry@math.ohio-state.edu), Dmitrii Pasechnik (dimpase@msri.org), Bruce Richter (brichter@math.uwaterloo.ca) and Gelasio Salazar* (gsalazar@cactus.iico.uaslp.mx).
Recent Progress on Turan's Brickyard Problem: Improved Lower Bounds for the Crossing Numbers of $K_{m, n}$ and $K_{n}$.
In the earliest instance of a crossing number problem, Turán conjectured in 1945 that the crossing number $c r\left(K_{m, n}\right)$ of $K_{m, n}$ is $\lfloor(m-1) / 2\rfloor\lfloor m / 2\rfloor\lfloor(n-1) / 2\rfloor\lfloor n / 2\rfloor$. By using some elementary topological arguments, we set up a quadratic optimization problem whose minimum yields a lower bound for $\operatorname{cr}\left(K_{m, n}\right)$. Although the quadratic problem is intractable because of its size, by using some very recent relaxation techniques for quadratic programming we were able to show that $c r\left(K_{m, n}\right)$ is at least 0.83 of its conjectured value, for each fixed $m$ and sufficiently large $n$. This also implies that the crossing number of the complete graph $K_{n}$ is asymptotically at least 0.83 of its long-conjectured value. (Received March 02, 2004)

