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Yongwei Yao* (ywyao@umich.edu), Department of Mathematics, University of Michigan, East Hall, 525 E University, Ann Arbor, MI 48109. Observations on the F-signature of local rings of characteristic p. Preliminary report.

Let (R, \mathfrak{m}, k) be a d-dimensional Noetherian reduced local ring of prime characteristic p such that R^{1/p^e} are finite over R for all $e \in \mathbb{N}$ (i.e. R is F-finite). Consider the sequence $\{\frac{a_e}{q^{\alpha(R)}+d}\}_{e=0}^{\infty}$, in which $\alpha(R) = \log_p[k:k^p]$, $q = p^e$, and a_e is the maximal rank of free R-modules appearing as direct summands of R-module $R^{1/q}$. Denote by $s^-(R)$ and $s^+(R)$ the liminf and limsup respectively of the above sequence as $e \to \infty$. If $s^-(R) = s^+(R)$, then the limit, denoted by s(R), is called the F-signature of R, which was first defined and studied by Huneke and Leuschke. It turns out that the F-signature can be defined in a way that is independent of the module finite property of $R^{1/q}$ over R. We show that: (1) If $s^+(R) \ge 1 - \frac{1}{d!p^d}$, then R is regular; (2) If R is excellent such that R_P is Gorenstein for every $P \in \operatorname{Spec}(R) \setminus \{\mathfrak{m}\}$, then s(R) exists; (3) If $(R,\mathfrak{m}) \to (S,\mathfrak{n})$ is a local flat ring homomorphism, then $s^{\pm}(R) \ge s^{\pm}(S)$ and, if furthermore $S/\mathfrak{m}S$ is Gorenstein, then $s^{\pm}(S) \ge s^{\pm}(R)s(S/\mathfrak{m}S)$. (Received February 15, 2004)