Meeting: 998, Houston, Texas, SS 1A, Special Session on Graph Theory and Combinatorics

998-05-374 **Xiao Chen**, Department of Computer Science, Texas State University-San Marcos, San Marcos, TX 78666, and **Jian Shen*** (js48@txstate.edu), Department of Mathematics, Texas State University-San Marcos, San Marcos, TX 78666. *On the Frame-Stewart Conjecture about the Towers of Hanoi.*

The multi-peg Towers of Hanoi problem consists of k pegs mounted on a board together with n disks of different sizes. Initially these disks are placed on one peg in order of size, with the largest on the bottom. The rules of the problem allow disks to be moved one at a time from one peg to another as long as a disk is never placed on top of a smaller disk. The goal of the problem is to transfer all the disks to another peg with the minimum number of moves, denoted $H(n,k)$.

In 1941, Frame and Stewart proposed a recursive algorithm scheme to solve the problem. However proving the optimality of the Frame-Stewart algorithm remains open.

Let $FS(n,k)$ be the minimum number of moves needed to solve the Towers of Hanoi problem using the Frame-Stewart algorithm. In this talk, we prove that

$$\log FS(n,k) = \log H(n,k) + \Theta(k + \log n) = (n(k-2)!)^{1/(k-2)} + \Theta(k + \log n),$$

where the logarithmic function $\log(\cdot)$ has base 2. In other words, for $n \gg k \geq 4$, we show that $FS(n,k)$ and $H(n,k)$ both have the same order of magnitude of $2^{(1\pm o(1))(n(k-2)!)^{1/(k-2)}}$:

$$FS(n,k) = 2^{(1\pm o(1))(n(k-2)!)^{1/(k-2)}} = H(n,k).$$

This provides the strongest evidence so far to support the Frame-Stewart conjecture. (Received March 02, 2004)