998-16-71 **Alberto Facchini*** (facchini@math.unipd.it), Alberto Facchini, Universita' di Padova, Dipartimento di Matematica Pura e Applicata, Via Belzoni 7, I-35131 Padova, Italy. *Direct sum decompositions of modules, almost trace ideals, and pullbacks of monoids.* Preliminary report.

For a ring R, let $V(R)$ denote the commutative monoid of all isomorphism classes of finitely generated projective R-modules. The Grothendieck group $K_0(R)$ is the enveloping group of $V(R)$. There is a natural pre-order on $K_0(R)$ whose positive cone is the image of $V(R)$ in $K_0(R)$. We show that a number of pullback diagrams appear naturally in the study of the pre-ordered Grothendieck group $K_0(R)$. There is a one-to-one correspondence between the set of all trace ideals of R and the set $\mathrm{Spec}(V(R))$ of all prime ideals of the commutative monoid $V(R)$. Every ideal I of R contains a greatest trace ideal $\mathrm{Tr}(I)$. For any ideal I of R, the canonical projection $p\colon R \to R/I$ induces a monoid homomorphism $V(p)\colon V(R) \to V(R/I)$. This passage of projective modules from a ring R to the factor ring R/I turns out to be particularly good when I/Tr(I)$iscontainedintheJacobsonradicalofthefactorring$R/ $\mathrm{Tr}(I)$. We call the ideals I with this property *almost trace ideals*. We generalize to arbitrary rings a result by Goodearl that describes the lattice of the directed convex subgroups of $K_0(R)$ making use of a suitable subset of $\mathrm{Spec}(V(R))$. The results presented will appear in a joint paper with Pere Ara (Barcelona). (Received February 02, 2004)