Meeting: 998, Houston, Texas, SS 4A, Special Session on Nonlinear Analysis

998-35-243 **Zhi-Qiang Wang*** (wang@math.usu.edu). *On the Improved Hardy Inequalities.*

We prove the weighted version of the Improved Hardy Inequality: Let $\Omega \subset\subset B_R(0) \subset \mathbb{R}^n$ with $N \geq 1$ and $a < \frac{N-2}{2}$. Then there exists $C = C(a, \Omega)$ such that for all $u \in C_0^\infty(\Omega)$,

$$\begin{aligned} &\int_\Omega |x|^{-2a}|\nabla u|^2 \, dx - \left(\frac{N-2-2a}{2}\right)^2 \int_\Omega |x|^{-2(a+1)}|u|^2 dx \\ &\geq C(a,\Omega) \int_\Omega \left(\ln \frac{R}{|x|}\right)^{-2} |x|^{-2a}|\nabla u|^2 dx. \end{aligned} \tag{1}$$

The inequality is sharp in the sense that the weight $\left(\ln \frac{R}{|x|}\right)^{-2}$ on the right hand isde can not be replaced by $g(x)\left(\ln \frac{R}{|x|}\right)^{-2}$ with $g(x) \geq 0$ satisfying $g(x) \to +\infty$ as $|x| \to 0$. For the un-weighted case this improves the recent results of Brezis-Vazguez and Vazguez-Zuazua. This is a joint work with Michel Willem. (Received February 29, 2004)