Meeting: 998, Houston, Texas, SS 3A, Special Session on Harmonic and Functional Analysis

998-46-429 **Stephen A. Saxon***. *New analyses and analogs of Warner boundedness.*

An important feature of many locally convex topological vector spaces is a fundamental sequence of bounded sets (fsbs); *e.g.*, all *df*-spaces and *DF*-spaces. Warner proved that a $T_{3\frac{1}{2}}$-space X is Warner bounded if and only if $C_c(X)$, the space of continuous real-valued functions on X endowed with the compact-open topology, has a fsbs. He gave several other equivalent conditions. We (Jerzy Kąkol, Aaron Todd and myself) give much simpler arguments by first showing that X is Warner bounded if and only if $C_c(X)$ does not contain a linear and topological copy of a dense subspace of the product space $\mathbb{R}^{\mathbb{N}}$. As a consequence, additional analytic characterizations become readily available. A number of their analogs very nicely characterize when X is pseudocompact, and when $C_c(X)$ is a *df*-space, the latter amply answering Jarchow's 1981 question. For example, we find that X is pseudocompact if and only if $C_c(X)$ does not contain a copy of $\mathbb{R}^{\mathbb{N}}$, and $C_c(X)$ is a *df*-space if and only if its strong dual is a Banach space. Another example: X is pseudocompact, X is Warner bounded, or $C_c(X)$ is a *df* -space if and only if for each sequence $(\mu_n)_n \subset C_c(X)'$ there exists a sequence $(\varepsilon_n)_n \subset (0,1]$ such that $(\varepsilon_n \mu_n)_n$ is weakly bounded, is strongly bounded, or is equicontinuous, respectively. In related work, we more than answer the 1973 Buchwalter-Schmets question by demonstrating a $C_c(X)$ space that is a *df*-space but not a *DF*-space. (Received March 09, 2004)