

998-62-63

Miguel A Arcones* (arcones@math.binghamton.edu), , Binghamton, NY 13902. *The large deviation principle for M-estimators*. Preliminary report.

We discuss the large deviation principle for M-estimators (and maximum likelihood estimators in particular). Under certain smooth conditions mle's satisfy the large deviation principle with speed n and rate function $I_\theta(t) := -\inf_{\lambda \in \mathfrak{R}^d} \ln E_\theta[\exp(\lambda' \nabla_t \ln f(X, t))]$, where $\{f(x, t) : t \in \Theta\}$ is a family of pdf's, $\Theta \subset \mathfrak{R}^d$ and ∇ denotes the gradient. We have that $I_\theta(t) \leq K(f(\cdot, t), f(\cdot, \theta))$, where $K(f(\cdot, t), f(\cdot, \theta))$ is the Kullback-Leibler information between $f(\cdot, t)$ and $f(\cdot, \theta)$. We determine the class of parametric for which $I_\theta(t) = K(f(\cdot, t), f(\cdot, \theta))$. For these parametric families the mle is exponentially efficient. If T_n is a consistent estimator of θ , for each $\theta \in \Theta$, then, for each $\theta \in \Theta$,

$$\liminf_{n \rightarrow \infty} n^{-1} \ln (P_\theta\{|T_n - \theta| > \epsilon\}) \geq -\inf\{K(f(\cdot, \theta_1), f(\cdot, \theta)) : \theta_1 \text{ satisfying } |\theta_1 - \theta| > \epsilon\}.$$

Whenever, the $I_\theta(t) = K(f(\cdot, t), f(\cdot, \theta))$, the mle minimizes the limit above over all possible estimators. (Received January 17, 2004)