

Meeting: 1000, Albuquerque, New Mexico, SS 13A, Special Session on Analysis and Geometry in Carnot-Carathéodory Spaces

1000-28-113 **Zoltan M. Balogh**, Department of Mathematics, University of Berne, Sidlerstrasse 5 CH-3012, Berne, Switzerland, and **Jeremy T. Tyson*** (tyson@math.uiuc.edu), Department of Mathematics, University of Illinois, 1409 W. Green St., Urbana, IL 61801. *Comparison theorems for Hausdorff dimension in the Heisenberg group.*

Motivation for this work comes from a question of Gromov, which asks for sharp comparison theorems for the Riemannian and sub-Riemannian Hausdorff measures on a sub-Riemannian manifold. We answer Gromov's question in the first Heisenberg group H . Let d_E , respectively d_{CC} , denote the Euclidean, respectively Carnot-Carathéodory, metric in H , and let $\beta_{\pm}(\alpha)$ be the supremum/infimum of the values of the CC Hausdorff dimensions $H - \dim_{CC}(A)$, taken over all subsets $A \subset H$ with Euclidean Hausdorff dimension $H - \dim_E(A) = \alpha$. Balogh, Rickly and Serra-Cassano have shown that

$$\beta_-(\alpha) = \max\{\alpha, 2\alpha - 2\}$$

and

$$\beta_+(\alpha) = \min\{2\alpha, \alpha + 1\}$$

for all $0 \leq \alpha \leq 3$, and that the supremum is achieved for all α and the infimum is achieved for all $\alpha \in [0, 2) \cup \{3\}$. We show that the infimum is achieved for all α . The relevant examples when $\alpha = 2$ are horizontal subsets of H parameterized by BV functions on planar domains. By way of contrast, the pure 2-unrectifiability of H (a recent result of Ambrosio and Kirchheim) proscribes Lipschitz horizontal surfaces. (Received August 20, 2004)