Meeting: 1001, Evanston, Illinois, SS 2A, Special Session on Extremal Combinatorics

1001-05-181 **Jacques A Verstraete*** (`jverstra@math.uwaterloo.ca`), Faculty of Mathematics, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada. *Extremal Problems for Linear Dependences.*

Let V be an n-dimensional vector space over a finite field F. A set $X \subset V$ of vectors of weight at most r is said to be *k-wise linearly independent* if any set of k vectors in X is linearly independent. Define

$$\rho(n,k,r) = \max\{|X| : X \subset V \text{ and } X \text{ is } k\text{-wise linearly independent}\}.$$

The problem of estimating $\rho(n,k,r)$ is directly related to finding sparse parity check matrices for linear codes with large minimum distance, and some extremal hypergraph problems. In this talk we present the first general bounds for the quantity $\rho(n,k,r)$. In particular, for each fixed r, we prove that

$$\lim_{k\to\infty} \frac{\log \rho(n,k,r)}{r \log n} \quad = \quad \frac{1}{2}.$$

This improves upon earlier results of Lefmann, Pudlák and Savický. We use our bounds to give an asymptotic solution to an old problem of Erdős in combinatorial number theory. (Received August 24, 2004)