Meeting: 1001, Evanston, Illinois, SS 2A, Special Session on Extremal Combinatorics

1001-05-403 **Evelin Toumpakari*** (evelint@math.uchicago.edu), 5734 S. University Ave, Chicago, IL 60637. *On the abelian sandpile model.*

Motivated by statistical physics (self-organized criticality), the abelian sandpile automaton is a variant of the chip-firing game on a rooted connected graph X. Every ordinary (non-root) vertex has an associated pile of identical grains. When the height h(i) of the pile at an ordinary vertex i exceeds deg(i)-1, i "topples", i.e., loses deg(i) grains, one to each neighbor. Grains passed to the root disappear; therefore, every toppling sequence ("avalanche") is finite. A state is "stable" if h(i) $<$ deg(i) for each ordinary vertex i. Lovasz at al showed that the stable state reached after an avalanche depends solely on the initial state. This permits to define addition on the set S of stable states, by adding pointwise and toppling. (S,+) is a commutative semigroup with a unique idempotent e. The ideal G:=e+S generated by e is a group, the "abelian sandpile group" of X. The elements of G are precisely the recurrent states of the Markov chain naturally associated with the model. The order of G is the number of spanning trees of X; the defining relations of G correspond to the rows of the Laplacian of X.

We study combinatorial, algebraic, and algorithmic aspects of this model. Some of the results are joint work with Laszlo Babai. (Received August 31, 2004)