Meeting: 1001, Evanston, Illinois, SS 2A, Special Session on Extremal Combinatorics

1001-05-429 **Gyula O.H. Katona*** (ohkatona@renyi.hu), Renyi Institute, Realtanoda u. 13-15, 1053 Budapest, Hungary. *Families with forbidden inclusion pattern.* Preliminary report.

Let X be a finite set and $\mathcal{F}$ be a family of its subsets. $\max|\mathcal{F}|$ is determined when certain configurations are excluded. The excluded configurations are determined by inclusions only. The following one is a typical theorem. Suppose that $\mathcal{F}$ contains no 4 distinct members A_1, A_2, B_1, B_2 such that $A_1, A_2 \subset B_1, B_2$ (4 inclusions). Then $|\mathcal{F}|$ is at most the size of the two largest levels, that is the number of all subsets of sizes $\lfloor \frac{n-1}{2} \rfloor$ and $\lceil \frac{n}{2} \rceil$. Another example is when the family contains no $r+1$ distinct members satisfying $A \subset B_1, \ldots, B_r$. Then the family can have at most $\binom{n}{\lfloor \frac{n}{2} \rfloor}(1 + 2\frac{r-1}{n} + o(\frac{1}{n}))$ members. This is nearly sharp, since there is a construction containing $\binom{n}{\lfloor \frac{n}{2} \rfloor}(1 + \frac{r-1}{n} + o(\frac{1}{n}))$ members. It is somewhat surprising that such types of asymptotical results can be obtained by the cycle method, used in the proofs.

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