Meeting: 1001, Evanston, Illinois, SS 16A, Special Session on Spectral Problems of Differential Operators

1001-34-393 **Kwang C. Shin*** (kcshin@math.missouri.edu), Department of Mathematics, University of Missouri, Columbia, MO 65211. *Asymptotic expansions of the eigenvalues of anharmonic oscillators.* Preliminary report.

We consider the following "nonstandard" Schrödinger-type eigenvalue problem

$$-\frac{d^2}{dz^2}u(\lambda,z)+P(z)u(\lambda,z)=\lambda u(\lambda,z), \tag{1}$$

where P is a polynomial of degree $m\geq 3$, under the boundary condition that $u(\lambda,z)\to 0$ as $z\to\infty$ along two rays in the complex plane.

In this talk, a proof of the following asymptotic expansion of the eigenvalues $\{\lambda_n\}_{n\in\mathbb{N}}$ of this boundary value problem will be sketched:

$$\lambda_n=\sum_{j=0}^{\lfloor\frac{m+2}{2}\rfloor}K_j n^{\frac{2m-2j}{m+2}}+o\left(n^{\frac{2m-2\lfloor\frac{m+2}{2}\rfloor}{m+2}}\right), \tag{2}$$

as $n\to\infty$, where every K_j for $0\leq j\leq\frac{m}{2}$ can be explicitly computed in terms of the degree and the coefficients of P and the boundary condition.

This asymptotic expansion implies reality of all but finitely many eigenvalues, in the $\mathcal{PT}$-symmetric setting.

Bender and Wu, Maslov, Sibuya, Birman and Solomyak, Helffer and Robert, and many others obtained asymptotic expansions of the eigenvalues, some of which have error terms superior than the one above, but with particular choices of the polynomial P and the boundary condition. (Received August 31, 2004)