Meeting: 1004, Bowling Green, Kentucky, SS 11A, Special Session on Commutative Ring Theory

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Schreier Property and Gauss' Lemma. Preliminary report.

Let D be an integral domain with quotient field K.Call D Schreier if D is integrally closed and for all $x, y, z \in D \backslash \{0\}$ $x \mid yz$ implies that $x = rs$ where $r \mid y$ and $s \mid z$. A GCD domain is Schreier. One aim of this talk is to show that an integral domain D is a GCD domain if and only if (i) for each pair $a, b \in D \backslash \{0\}$, there is a finitely generated ideal B such that $aD \cap bD = B_v = (B^{-1})^{-1}$ and (ii) every quadratic in $D[X]$ that is a product of two linear polynomials in $K[X]$ is a product of two linear polynomials in $D[X]$. We also show that D is Schreier if and only if every polynomial in $D[X]$ with a linear factor in $K[X]$ has a linear factor in $D[X]$ and show that D is a Schreier domain with algebraically closed field of fractions if and only if every nonconstant polynomial over D is expressible as a product of linear polynomials. We also compare the two most common modes of generalizing GCD domains. One is via properties that imply Gauss' Lemma and the other is via the Schreier property. The Schreier property is not implied by any of the specializations of Gauss' Lemma while all but one of the specializations of Gauss Lemma are implied by the Schreier property. (Received January 19, 2005)