

Meeting: 1004, Bowling Green, Kentucky, SS 11A, Special Session on Commutative Ring Theory

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01602-2597. *Idealization.*

Let R be a commutative ring and M an R -module. The idealization $R(+)M$ of M in R is given by $R(+)M = \{(r, m) | r \in R, m \in M\}$. If (r, m) and (s, n) are two elements of $R(+)M$, we define a) $(r, m) = (s, n)$ if $r = s$ and $m = n$, b) $(r, m) + (s, n) = (r + s, m + n)$, and c) $(r, m)(s, n) = (rs, rn + sm)$. With these definitions $R(+)M$ becomes a commutative ring with identity. In this talk we survey known results about $R(+)M$ and give some new ones. Ideals of $R(+)M$, especially those of the form $I(+)C$, where I is an ideal of R and C is a submodule of M , are studied. Certain distinguished elements of $R(+)M$ are also found. Conditions of R and M are determined to make $R(+)M$ Noetherian, Artinian, a valuation ring, a chained ring, a PIR, and a graded ring. We also define a functor from the category of R -modules to the category of R -algebras given by $F(M) = R(+)M$. (Received January 20, 2005)