The 3-colored Ramsey Number of Odd Cycles.

For graphs $L_1, \ldots, L_k$, the Ramsey number $R(L_1, \ldots, L_k)$ is the minimum integer $N$ satisfying that for any coloring of the edges of the complete graph $K_N$ on $N$ vertices by $k$ colors there exists a color $i$ for which the corresponding color class contains $L_i$ as a subgraph.

In 1973, Bondy and Erdős conjectured that if $n$ is odd and $C_n$ denotes the cycle on $n$ vertices, then $R(C_n, C_n, C_n) = 4n - 3$. In 1999, Łuczak proved that $R(C_n, C_n, C_n) = 4n + o(n)$, where $o(n)/n \to 0$ as $n \to \infty$. In this paper we strengthen Łuczak’s result and verify this conjecture for $n$ sufficiently large. (Received February 08, 2005)