Meeting: 1005, Newark, Delaware, SS 9A, Special Session on Arithmetic Groups and Related Topics

1005-20-126 **Peter Abramenko*** (pa8e@virginia.edu), Department of Mathematics, P.O. Box 400137, University of Virginia, Charlottesville, VA 22904. *Non-finitely generated groups between $SL_2(R)$ and $SL_2(K)$.*

Andrei Rapinchuk raised the following question: Given an integral domain R, finitely generated as a $\mathbb{Z}$-algebra, with field of fractions K, and a reductive group G defined over K, is it possible to find a finitely generated subgroup of $G(K)$ which contains $G(R)$? For S-arithmetic groups the answer is always yes (by results of Borel and Harish-chandra in characteristic 0 and of Behr in characteristic $p > 0$). There is also some evidence pointing to a positive answer for $G = SL_n$ and n "big enough", for instance $n > d + 1$, where d is the Krull dimension of R. However, for $G = SL_2$, we have the following
Theorem: *If R_0 is an integral domain with infinitely many non-associate prime elements and $R = R_0[s, t]$ (polynomial ring in 2 variables), then any subgroup of $SL_2(K)$ containing $SL_2(R)$ is not finitely generated.*

The proof uses the action of $SL_2(K)$ on suitable Bruhat-Tits trees and some facts concerning subgroups of $SL_2(K)$ generated by (finitely many) elementary matrices. Some anticipated generalizations of the theorem will be indicated as well.

(Received February 03, 2005)