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48109-1109. *Singularities and complex dynamics.*

I will discuss how algebro-geometric methods can sometimes be used to study objects of nonalgebraic nature, e.g. certain dynamical systems.

In dynamics one is often interested in asymptotic behavior as time evolves. For instance, given a polynomial map $F : C^2 \rightarrow C^2$ one may ask at what speed the orbit $p, F(p), F(F(p)), \dots, F^n(p)$ approaches infinity as n increases, if the original point p is chosen generically near infinity. This speed is governed by the behavior of $\deg(F^n)$, the degree of the highest order term in F^n . For example, if $F(X, Y) = (Y, XY)$, then $\deg(F^n)$ gives the Fibonacci numbers, so in a suitable sense, the speed above equals the golden mean.

A classical field of algebraic geometry is the study of singularities, such as the curve in C^2 parameterized by $t \rightarrow (t^2, t^3)$, which has a cusp at the origin. It is known that singularities typically can be resolved, i.e. viewed as “shadows” of nonsingular objects; the cusp above is the shadow of the space curve $t \rightarrow (t, t^2, t^3)$.

As I will explain, it turns out that a dynamic version of resolution of curve singularities can be used to understand the speed of convergence to infinity of polynomial maps of C^2 . As a consequence, the speed is always a quadratic integer. (Received April 30, 2004)