Meeting: 1006, Lubbock, Texas, SS 5A, Special Session on Recent Advances in Complex Function Theory

1006-30-237 **Jeremy T. Tyson*** (tyson@math.uiuc.edu), Department of Mathematics, University of Illinois, 1409 West Green St., Urbana, IL 61801. *Smale's Mean Value Conjecture for Complex Polynomials.*

Smale's Mean Value Conjecture is one of the major outstanding problems in the geometry of complex polynomials. Motivated by computational issues associated with the use of Newton's method, Smale (1981) proved that

$$\min_w \left| \frac{P(w)}{wP'(0)} \right| \leq K$$

for every degree d polynomial P, $P(0) = 0$, $P'(0) \neq 0$, where the infimum is taken over all critical points w of P and $K = 4$. He conjectured that one could choose $K = 1 - \frac{1}{d}$. The example $P_0(z) = z^d - dz$ shows that this value for K, if true, would be sharp.

Tischler (1989) proposed the following strong form of Smale's conjecture: for all P as above,

$$\min_w \left| \frac{1}{2} - \frac{P(w)}{wP'(0)} \right| \leq K_1$$

where $K_1 = \frac{1}{2} - \frac{1}{d}$. As evidence, Tischler proved his conjecture for small perturbations of P_0, and in the case $d \leq 4$.

We prove Tischler's conjecture for small perturbations of $P_1(z) = (z-1)^d - (-1)^d$, but we construct counterexamples in each degree $d \geq 5$. We then prove estimates for weighted L^2-averages of the Smale values $P(w_j)/w_j$ of a degree d polynomial P with critical points w_j, $j = 1, \ldots, d-1$, which yield the best current values for K and K_1 when $5 \leq d \leq 7$. (Received February 15, 2005)