Meeting: 1007, Santa Barbara, California, AMS CP 1, Session for Contributed Papers

1007-11-61 **Hung-ping Tsao*** (hptsao@hotmail.com), 1151 Highland Drive, Novato, CA 94949. *Sums of Powers of Arithmetic Progressions.*

The main purpose is to find the polynomial expression in n for the sum of the first n kth powers of an arithmetic progression with the leading term a and the difference d. By considering such sum S(k) to be the kth power of S, we can define the linear factorization of a "polynomial" in S by way of that of ordinary polynomials. For example, (S-a)(S-a-d) denotes S(2)-(2a+d)S(1)+a(a+d)S(0), which can further be denoted as P(S-a,2;d), where P(n,r;d)=n(n-d)(n-2d)...[n-(r-1)d]. Then we can prove the following theorem by using the mathematical induction on n.

Theorem. P(dn,r;d)=rdP(S-a,r-1;d).

As a result, the permutation P(n,r) can be expressed as a linear combination of S(k)'s with the coefficients being functions of a, d and r(binomial coefficients and Stirling's numbers of the first kind involving r), from which the polynomial expression for each S(k) can be obtained explicitly. As a special case, the sum of the first n kth powers of the natural numbers can be expressed as a polynomial in n with the coefficients involving Stirling's numbers of both kinds. As a side product, we have a formula to construct Stirling's numbers of the second kind from the first kind. (Received January 26, 2005)