

Meeting: 1007, Santa Barbara, California, SS 8A, Special Session on Geometry and Physics

1007-58-86 **Philippe Souplet** and **Qi S. Zhang*** (qizhang@math.ucr.edu), Riverside, CA 92521. *Sharp gradient estimate and Yau's Liouville theorem for the heat equation on noncompact manifolds.*

We derive a sharp, localized version of elliptic type gradient estimates for positive solutions (bounded or not) to the heat equation. These estimates are akin to the Cheng-Yau estimate for the Laplace equation and Hamilton's estimate for bounded solutions to the heat equation on compact manifolds. As applications, we generalize Yau's celebrated Liouville theorem for positive harmonic functions to positive eternal solutions of the heat equation, under certain growth condition. Surprisingly, this Liouville theorem for the heat equation does not hold even in \mathbf{R}^n without such a condition. We also prove a sharpened long time gradient estimate for the log of heat kernel on noncompact manifolds. This has been an open problem in view of the well known estimates in the compact, short time case. (Received February 03, 2005)