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We present a new 'class' of models for image restoration and decomposition based on the Total-Variation-Minimization framework of Rudin-Osher-Fatemi. In our model, the data image  $f$  is decomposed into the sum of  $u$  and  $v = f - u$ , where  $u$  is the piecewise-smooth (cartoon) component and  $v$  the oscillatory (noisy and textured) part of  $f$ . Motivated by Y. Meyer's suggestions to model the  $v$  component with norms weaker than the  $L^2$  norm, by Osher-Solé-Vese ( $BV, H^{-1}$ ) model, and by Mumford-Gidas's remark that Gaussian white noise is supported in  $\cap_{\epsilon>0} H_{loc}^{-1-\epsilon}$ , we impose in our model that  $u$  be in the space of bounded variation  $BV(\Omega)$  and  $v$  be in the 'class' of negative Hilbert-Sobolev spaces  $H^{-s}(\mathbb{R}^2)$ , for  $s > 0$ . Under these constraints, our derived energy functional is strictly convex, hence existence and uniqueness of solution is guaranteed. When  $s = 0$ , our model reduces to the  $(BV, L^2)$  decomposition of Rudin, Osher, and Fatemi. We present a numerical algorithm for computing the  $H^{-s}$  norm for images, as well as for computing the minimizer of the energy functional in our model. We also give the definition for a semi-norm  $|\cdot|_*$  with which we are able to give characterizations of minimizers. (Received August 15, 2005)