1009-20-54 **Ilya Kapovich**, **Igor Rivin**, **Paul Schupp** and **Vladimir Shpilrain*** (shpil@groups.sci.ccny.cuny.edu). *Asymptotic density in free and free abelian groups.*

Let F_k be the free group of finite rank $k \geq 2$ and let α be the abelianization map from F_k onto $\mathbb{Z}^k$. We prove that if a set $S \subseteq \mathbb{Z}^k$ is invariant under the natural action of $SL(k, \mathbb{Z})$ then the asymptotic density of S in $\mathbb{Z}^k$ and the asymptotic density of its full preimage $\alpha^{-1}(S)$ in F_k are equal. This implies, in particular, that for every integer $t \geq 1$, the asymptotic density of the set of elements in F_k that map to t-th powers of primitive elements in $\mathbb{Z}^k$ is equal to $\frac{1}{t^k \zeta(k)}$, where ζ is the Riemann zeta-function.

As an application of this result we show that the union of all proper retracts in the free group of rank two has asymptotic density $\frac{6}{\pi^2}$. This contrasts with the fact that the union of all proper free factors has asymptotic density 0. (Received July 31, 2005)