1009-33-118 **M. R. Hoare** and **Mizan Rahman*** (MRAHMAN@MATH.CARLETON.CA), School of Mathematics and Statistics, Carleton University, 4302 Herzberg Building, Ottawa, Ontario K1S 5B6, Canada.
Probabilistic Origin of a New System of Orthogonal Polynomials in 2 Variables: a Limit Case of the $9-j$ Symbols.

A brief sketch of a 2-variable extension of a special case of a Markov model introduced by Cooper, Hoare and Rahman (1977) is given, that raises the question of the eigenvalues and eigenfunctions of the transition kernel:

$$K(i_1,i_2;j_1,j_2) = \sum_{k_1}\sum_{k_2} b(k_1,i_1;\alpha_1)\ b(k_2,i_2;\alpha_2) \tag{1}$$

$$\times b_2(j_1-k_1,j_2-k_2;N-k_1-k_2;\beta_1,\beta_2), \tag{2}$$

where

$$b(k,i;\alpha) = \tbinom{i}{k}\alpha^k(1-\alpha)^{i-k}, \tag{3}$$

$$b_2(j,k;N;\beta_1,\beta_2) = \tfrac{N!}{j!k!(N-j-k)!}\ \beta_1^j\beta_2^k(1-\beta_1-\beta_2)^{N-j-k}. \tag{4}$$

We find that $b_2(x,y;N;\eta_1,\eta_2)$ times the polynomial

$$P_{m,n}(x,y) = \sum_i\sum_j\sum_k\sum_\ell \tfrac{(-m)_{i+j}(-n)_{k+\ell}(-x)_{i+k}(-y)_{j+\ell}}{i!j!k!\ell!(-N)_{i+j+k+\ell}} \tag{5}$$

$$\times t^i u^j v^k w^\ell \tag{6}$$

are the eigenfunctions of K, where t, u, v, w depend on the α's and β's in a nonlinear way, and

$$\frac{(1-\alpha_1)\eta_1}{\beta_1} = \frac{(1-\alpha_2)\eta_2}{\beta_2} = \Big(\frac{\beta_1}{1-\alpha_1}+\frac{\beta_2}{1-\alpha_2}+1-\beta_1-\beta_2\Big)^{-1}.$$

These functions are discovered as a Krawtchouk limit of Wigner's $9-j$ symbols. (Received August 10, 2005)