

1009-37-105

Marcus Pivato* (pivato@xaravve.trentu.ca), Department of Mathematics, Trent University,
Peter Gzowski College (Enweying), 2151 East Bank Drive, Peterborough, Ontario K9L 1Z8,
Canada. *Crystallographic Defects in Cellular Automata.*

Let $A^{\mathbb{Z}^D}$ be the Cantor space of \mathbb{Z}^D -indexed configurations in a finite alphabet A , and let σ be the \mathbb{Z}^D -action of shifts on $A^{\mathbb{Z}^D}$. A *cellular automaton* is a continuous, σ -commuting self-map Φ of $A^{\mathbb{Z}^D}$, and a Φ -invariant subshift is a closed, (Φ, σ) -invariant subset $X \subset A^{\mathbb{Z}^D}$. Suppose $x \in A^{\mathbb{Z}^D}$ is X -admissible everywhere except for a small region we call a *defect*. It is empirically known that such defects persist under iteration of Φ , and propagate like ‘particles’ which coalesce or annihilate when they collide. We construct algebraic invariants for these defects, which explain their persistence under Φ , and partly explain the outcomes of their collisions. Some invariants are based on the spectrum or cocycle-structure of X ; others arise from the higher-dimensional (co)homology/homotopy groups of X , generalizing methods of Conway, Lagarias, Geller, and Propp. We also study the motion of defect particles (in the case $D = 1$), and show that it falls into several regimes, ranging from simple deterministic motion, to random walks, to the emulation of Turing machines or pushdown automata. (Received August 09, 2005)