We prove that a set of \( n \) unit balls in \( \mathbb{R}^d \) admits at most four distinct geometric permutations when \( n \) is sufficiently large, thus settling a long-standing conjecture in combinatorial geometry. Our results were subsequently improved by Cheong, Goaoc and Na to two if \( n \geq 9 \) and three otherwise.

The constant bound for unit balls significantly improves upon the \( \Theta(n^{d-1}) \) bound for balls of arbitrary radii. Intrigued by this large gap between the two bounds, we also prove a bound of \( O(\gamma \log \gamma) \) on the geometric permutations of balls when the radius ratio between the largest to smallest balls is bounded by \( \gamma \), and a tight bound of \( 2^{d-1} \) on the geometric permutations of \( n \) disjoint rectangular boxes in \( \mathbb{R}^d \). (Received August 09, 2005)