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Warren Wm McGovern* (warrenb@bgnet.bgsu.edu), Department of Mathematics and Statistics, Bowling Green State University, Bowling Green, OH 43403. *Neat Rings*.

A ring in which every element is the sum of a unit and an idempotent is called a *clean ring*. The study of clean rings began in 1976 and has picked up in the last 10-15 years. Several fundamental results about clean rings include that a product of rings is clean if and only if each factor is clean, every local ring is clean, and that every homomorphic image of a clean ring is clean. It is this last property that leads us to neat rings.

A ring is called *neat* if every proper homomorphic image is a clean ring. The standard example of a neat ring is \mathbb{Z} , the ring of integers. In this presentation we will explore some elementary properties of neat rings and quickly turn to the study of commutative neat Bezout domains. It will be shown that within this class of domains the neat rings can be characterized using the domain's lattice-ordered group of divisibility. We will make use of Johnstone's Theorem which states that a commutative ring is a clean ring if and only if it is a pm-ring, that is, every prime ideal is contained in a unique maximal ideal, and its maximal ideal structure space is zero-dimensional (with respect to the Zariski topology). (Received June 25, 2005)