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**Ayman R Badawi\*** (abadawi@ausharjah.edu), American Univ. of Sharjah, Dept. of Math., P.O. Box 26666, 00000 Sharjah, United Arab Emirates, and **Ali Jaballah**, Univ. of Sharjah, United Arab Emirates. *Some finiteness conditions on the set of overrings of a  $\phi$ -ring*. Preliminary report.

Let  $\mathcal{H} = \{R \mid R \text{ is a commutative ring and } Nil(R) \text{ is a divided prime ideal of } R\}$ . For a ring  $R \in \mathcal{H}$  with total quotient ring  $T(R)$ , the map  $\phi : T(R) \longrightarrow R_{Nil(R)}$  such that  $\phi(a/b) = a/b$  for  $a \in R$  and a non-zero-divisor  $b$  of  $R$  is a ring homomorphism from  $T(R)$  into  $R_{Nil(R)}$ , and  $\phi$  restricted to  $R$  is also a ring homomorphism from  $R$  into  $R_{Nil(R)}$  given by  $\phi(x) = x/1$  for every  $x \in R$ . An integral domain  $R$  is said to be an FC-domain (in the sense of Jaballah and Gilmer) if each chain of distinct overrings of  $R$  is finite, and  $R$  is called an FO-domain if  $R$  has finitely many overrings. A ring  $R$  is called an *FC-ring* if each chain of distinct overrings of  $R$  is finite, and  $R$  is said to be an *FO-ring* if  $R$  has finitely many overrings. A ring  $R \in \mathcal{H}$  is said to be a  $\phi$ -FC-ring if  $\phi(R)$  is an FC-ring, and  $R$  is called a  $\phi$ -FO-ring if  $\phi(R)$  is an FO-ring. In this paper, we show that the theory of  $\phi$ -FC-rings and  $\phi$ -FO-rings resembles that of FC-domains and FO-domains. (Received July 06, 2005)