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Let $k \in \{1, \dots, n\}$. The k -numerical range of $A \in M_n$ is the set

$$W_k(A) = \{(\operatorname{tr} X^*AX)/k : X \text{ is } n \times k, X^*X = I_k\},$$

and the k -numerical radius of A is the quantity

$$w_k(A) = \max\{|z| : z \in W_k(A)\}.$$

Suppose $k > 1$, $k' \in \{1, \dots, n'\}$ and $n' < C(n, k) \min\{k', n' - k'\}$. It is shown that there is a linear map $\phi : M_n \rightarrow M_{n'}$ satisfying $W_{k'}(\phi(A)) = W_k(A)$ for all $A \in M_n$ if and only if $n'/n = k'/k$ or $n'/n = k'/(n - k)$ is a positive integer. Moreover, if such a linear map ϕ exists, then there are unitary matrix $U \in M_{n'}$ and nonnegative integers p, q with $p + q = n'/n$ such that ϕ has the form

$$A \mapsto U^* \left[\underbrace{A \oplus \dots \oplus A}_p \oplus \underbrace{A^t \oplus \dots \oplus A^t}_q \right] U$$

or

$$A \mapsto U^* \left[\underbrace{\psi(A) \oplus \dots \oplus \psi(A)}_p \oplus \underbrace{\psi(A)^t \oplus \dots \oplus \psi(A)^t}_q \right] U,$$

where $\psi : M_n \rightarrow M_n$ has the form $A \mapsto [(\operatorname{tr} A)I_n - (n - k)A]/k$. Linear maps $\tilde{\phi} : M_n \rightarrow M_{n'}$ satisfying $w_{k'}(\tilde{\phi}(A)) = w_k(A)$ for all $A \in M_n$ are also studied. Furthermore, results are extended to triangular matrices. (Received August 26, 2005)