1011-20-82 **Ilya Kapovich*** (`kapovich@math.uiuc.edu`), UIUC Department of Mathematics, 1409 West Green Street, Urbana, IL 61801, and **Igor Rivin**, **Paul E Schupp** and **Vladimir Shpilrain**.
Asymptotic density in free groups and $\mathbb{Z}^k$, Visible Points and Test Elements.

Let F_k be the free group of finite rank $k \geq 2$ and let α be the abelianization map from F_k onto $\mathbb{Z}^k$. We prove that if $S \subseteq \mathbb{Z}^k$ is invariant under the natural action of $SL(k, \mathbb{Z})$ then the asymptotic density of S in $\mathbb{Z}^k$ and the asymptotic density of its full preimage $\alpha^{-1}(S)$ in F_k are equal. This implies, in particular, that for every integer $t \geq 1$, the asymptotic density of the set of elements in F_k that map to t-th powers of primitive elements in $\mathbb{Z}^k$ is equal to to $\frac{1}{t^k \zeta(k)}$, where ζ is the Riemann zeta-function.

An element g of a group G is called a *test element* if every endomorphism of G which fixes g is an automorphism of G. As an application of the result above we prove that the asymptotic density of the set of all test elements in the free group $F(a, b)$ of rank two is $1 - \frac{6}{\pi^2}$. (Equivalently, this shows that the union of all proper retracts in $F(a, b)$ has asymptotic density $\frac{6}{\pi^2}$.) Thus being a test element in $F(a, b)$ is an "intermediate property" in the sense that the probability of being a test element is strictly between 0 and 1. (Received August 15, 2005)