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The Kadison-Singer Problem, posed in 1959, asks whether every pure state on the diagonal of  $B(\ell^2)$  extends uniquely to a pure state on all of  $B(\ell^2)$ . It is equivalent to a variety of important problems in mathematics and engineering (cf. Casazza's invited address at GPOTS 2005). Among these reformulations is the Paving Conjecture of Anderson:

*Given  $\varepsilon > 0$ , there exists a  $k \in \mathbb{N}$  such that for any  $n \in \mathbb{N}$  and any  $A \in M_n(\mathbb{C})$  with zero diagonal, there exist diagonal projections  $P_1, P_2, \dots, P_k \in M_n(\mathbb{C})$  such that  $P_1 + P_2 + \dots + P_k = I$  and*

$$\|P_j A P_j\| \leq \varepsilon \|A\|, \quad 1 \leq j \leq k.$$

In spite of significant progress by Berman-Halpern-Kaftal-Weiss and Bourgain-Tzafriri, the Paving Conjecture (and therefore the Kadison-Singer Problem) remain open. In this talk, we examine the Paving Conjecture for “small” parameter values:  $k = 3$  and  $n \leq 10$ . Using a combination of graph theory and operator theory, we determine the sharp  $\varepsilon$  for  $n = 4, 5, 6$ . By altogether different considerations, we produce examples of “bad pavers” of size  $n = 7, 10$ . (Received August 29, 2005)