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Shelling totally nonnegative flag varieties.

In this talk we will discuss the partially ordered set Q^J of cells in Rietsch's cell decomposition of the totally nonnegative part of an arbitrary flag variety $P_{\geq 0}^J$. In the case of the complete flag variety, this poset is the interval poset of the Bruhat order, and in the case of the (type A) Grassmannian, this poset is the cyclic Bruhat order introduced by Postnikov. Our goal is to understand the geometry of $P_{\geq 0}^J$: Lusztig has proved that this space is contractible, but it is unknown whether $P_{\geq 0}^J$ is homeomorphic to a ball. In addition, the topology of the individual cells is not well understood.

The order complex $\|Q^J\|$ is a simplicial complex which can be thought of as a combinatorial approximation of $P_{\geq 0}^J$. Using combinatorial tools such as Bjorner's EL-labellings and Dyer's reflection orders, we prove that Q^J is thin and EL-shellable. In particular, this implies that $\|Q^J\|$ is homeomorphic to a ball, and that Q^J is the face poset of some regular CW complex. Our results imply that *if* Rietsch's cell decomposition is regular, then $P_{\geq 0}^J$ is in fact homeomorphic to a ball. (Received July 20, 2005)