

1012-15-59

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A linear matrix is a matrix whose entries are linear multivariate polynomials of degree 0 or 1 over some (commutative) field k . This matrix may be considered as an element of the function field over k , and also as an element of the (noncommutative) free field over k . This leads to two notions of rank for the same linear matrix: the commutative rank and the noncommutative rank. The latter is smaller or equal to the former. To a linear matrix may also be associated a space of matrices over k , by giving to the variables all possible values in k ; the space (and the linear matrix) is by definition of low rank if the maximum rank of its elements is smaller than the number of rows and of columns. We show that for the given linear matrix, supposed to be of low rank, the commutative rank and the noncommutative rank coincide if and only if the associated subspace of matrices is a so-called compression space (which means roughly speaking that one may by change of bases over k on row and columns bring the matrix into a form having a big rectangle of 0's which is responsible for the commutative rank). The article is born from an attempt of the authors to give an efficient algorithm to compute the rank of a linear matrix (hence of any matrix) over the free field. (Received September 01, 2005)