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Ioan Bejenaru* (bejenaru@math.ucla.edu) and **Terence Tao**. *Sharp well-posedness and ill-posedness results for a quadratic non-linear Schrödinger equation.*

We establish that the quadratic non-linear Schrödinger equation

$$iu_t + u_{xx} = u^2$$

where $u : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{C}$, is locally well-posed in $H^s(\mathbb{R})$ when $s \geq -1$ and ill-posed when $s < -1$. Previous work had established local well-posedness for $s > -3/4$. The local well-posedness is achieved by an iteration using a modification of the standard $X^{s,b}$ spaces. The ill-posedness uses an abstract and general argument relying on the high-to-low frequency cascade present in the non-linearity, and a computation of the first non-linear iterate. (Received September 12, 2005)